

10F MAT2705-06/02 TEST 1 Answers 1

① a) $x(t) = 2 \Leftrightarrow t = 1, x = 2$

b) $\frac{dx}{dt} = \frac{4t - 3x}{t} = 0 \rightarrow \boxed{x = \frac{4}{3}t}$

c) not separable, is linear so:

$$\frac{dx}{dt} = 4 - \frac{3}{t}x$$

$$t^3 \left[\frac{dx}{dt} + \frac{3}{t}x = 4 \right]$$

$$\int \frac{3}{t} dt = 3 \ln t \quad (t > 0)$$

$$e^{\int \frac{3}{t} dt} = t^3$$

$$\frac{d}{dt}(xt^3) = 4t^3$$

$$xt^3 = \int 4t^3 dt = 4\left(\frac{t^4}{4}\right) + C$$

$$= t^4 + C$$

$$\boxed{x = \frac{t^4 + C}{t^3} = t + \frac{C}{t^3}}$$

d) $2 = x(1) = 1 + \frac{C}{1^3} = 1 + C$

$$\rightarrow C = 2 - 1 = 1$$

$$\boxed{x = t + \frac{1}{t^3}}$$

e) $\frac{dx}{dt} = \frac{d}{dt}(t + t^{-3}) = 1 - 3t^{-4} = 0$

$$1 = 3t^{-4}$$

$$t^4 = 3, t = \boxed{3^{1/4} \approx 1.32}$$

$$x = 3^{1/4} + \frac{1}{(3^{1/4})^3} = 3^{1/4} + \frac{1}{3^{3/4} 3^{1/4}}$$

$$= 3^{1/4} + \frac{3^{1/4}}{3} = \boxed{\frac{4}{3} 3^{1/4} \approx 1.75}$$

(= $\frac{4}{3}t$)

f) It should lie on the isocline $x = \frac{4}{3}t$ which it does.

g) Maple gives the expanded fraction form $x = t + \frac{1}{t^3}$.

② a) $\frac{dy}{dx} = (2x-2)(2-y)$

separable & linear
separable method easier

$$\int \frac{dy}{-(y-2)} = \int (2x-2) dx$$

$$-\ln|y-2| = 2\left(\frac{x^2}{2}\right) - 2x + C_1$$

$$\ln|y-2| = -x^2 + 2x - C_1$$

$$e^{\ln|y-2|} = e^{-x^2 + 2x - C_1}$$

$$|y-2| = e^{-C_1} e^{-x^2 + 2x}$$

$$y-2 = \underbrace{\pm e^{-C_1}}_C e^{-x^2 + 2x}$$

$$\boxed{y = 2 + C e^{-x^2 + 2x}}$$

b) $5 = y(2) = 2 + C e^{-4+4} = 2 + C$

$$C = 5 - 2 = 3$$

$$\boxed{y = 2 + 3 e^{-x(x-2)} = 2 + 3 e^{-x^2 + 2x}}$$

c) $\lim_{x \rightarrow \infty} y = \lim_{x \rightarrow \infty} (2 + 3 e^{-x^2 + 2x})$

$$= 2 + 3 \lim_{x \rightarrow \infty} \underbrace{e^{-x^2 + 2x}}_{\rightarrow -\infty} = 2.$$

$$2 + 3 e^{-x^2 + 2x} = 2.02$$

$$3 e^{-x^2 + 2x} = 0.02$$

$$e^{-x^2 + 2x} = 0.02/3$$

$$-x^2 + 2x = \ln 0.02/3$$

$$x^2 - 2x + \ln 0.02/3 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4 \ln 0.02/3}}{2} = 1 \pm \sqrt{1 + \ln(3/0.02)}$$

$$= 1 \pm \sqrt{1 + \ln 150} \approx -1.4817, 3.4517$$

↑ sign for $x > 1$.

$$\boxed{x \approx 3.45}$$

$$\boxed{x = 1 + \sqrt{1 - \ln 0.02/3} = 1 + \sqrt{1 + \ln 150}}$$

① a), b), e)

