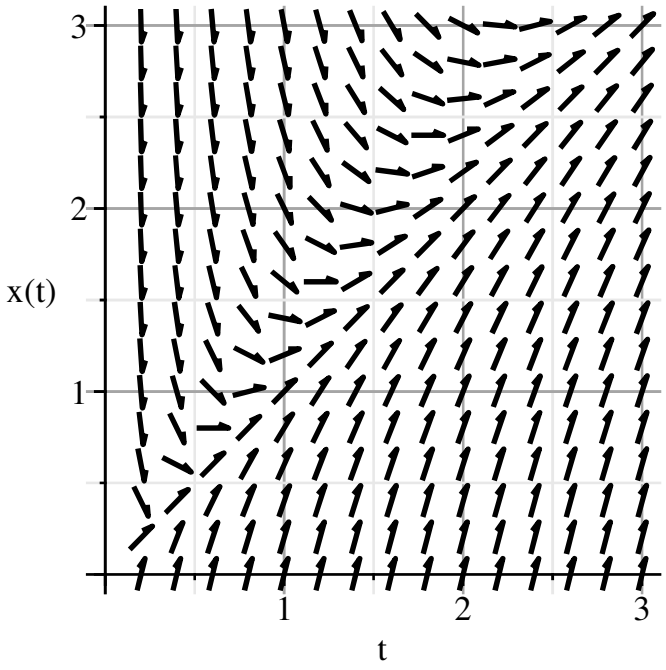
MAT2705-02/06 10F Test 1 Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $t\,\frac{dx}{dt} + 3x - 4t = 0\,,\ x(1) = 2\,,\quad t > 0\,.$

a) Hand draw in the solution of this differential equation satisfying the initial condition on the associated direction field to the right. Put a circled dot at the point corresponding to the initial condition.
b) What is the equation of the isocline representing all points where the slope field has slope zero? Draw in this curve.
c) Find the general solution of this differential equation by hand. Simplify it and box it.
d) Find the solution of this differential equation which satisfies the given initial condition. Simplify it and box it.
e) Find the exact values of t and x where your IVP solution curve has its minimum, and their approximate values to 2 decimal places. Locate this point on your curve with a squared dot.
f) Is your point consistent with part b)? Explain.
g) Optional: Does your initial value problem solution agree with Maple? If not, what does Maple produce? [To retain t as the independent variable, use function notation: $x(t)$ and $x'(t)$ for the unknown and its derivative.]



2. $\frac{dy}{dx} + (2-y)(2-2x) = 0\,,\ y(2) = 5$

a) Find the general solution of this differential equation by hand. Simplify it and box it.
b) Find the solution of this differential equation which satisfies the given initial condition. Simplify it and box it.
c) Notice that this differential equation has an equilibrium solution at $y = 2$. Show that $\lim_{x\to\infty} y = 2$ holds for your solution of part b). How large does $x > 2$ have to become before y decreases to the value 2.02 (i.e., to within 1 percent of the asymptotic value)? Give this value X exactly and to 2 decimal places.
d) Make a rough sketch of the solution curve and its asymptote in an appropriate window, labeling the axes and tickmarks and put a vertical line at $x = X$. Is your sketch consistent with part c)? Explain.

▶ solution