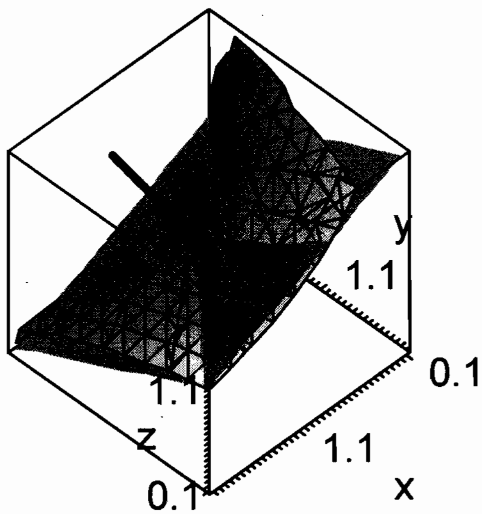
MAT2500-03/04 09S Quiz 6 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of each problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).



1. $f(x, y, z) = x^2 y z^3 - x y^2 z$

a) Write an equation for the level surface of this function passing through (1,1,1).

b) Evaluate the gradient vector field and its value at (1,1,1), identifying it with its proper symbol.

c) Write an equation for the normal line to the level surface through (1,1,1). At what point $\langle x_0, y_0, z_0 \rangle$ does the normal line intersect the x-y plane?

d) Write an equation for the tangent plane to this level surface at (1,1,1).

e) What is the directional derivative of f at (1,1,1) in the direction of the point (2,3,3)? Identify it by its proper symbol. Is the function increasing or decreasing in this direction?

▶ solution

a) $f(1,1,1) = 1-1 = 0$ so:

$\boxed{x^2yz^3 - xy^2z = 0}$

b) $\frac{\partial f}{\partial x} = (2x)yz^3 - y^2z$

$\frac{\partial f}{\partial y} = x^2z^3 - x(2y)z$

$\frac{\partial f}{\partial z} = x^2y(3z^2) - xy^2$

$\vec{\nabla} f(x,y,z) = \boxed{\langle 2xyz^3 - y^2z,\ x^2z^3 - 2xyz,\ 3x^2yz^2 - xy^2 \rangle}$

$\vec{\nabla} f(1,1,1) = \langle 2-1, 1-2, 3-1 \rangle = \boxed{\langle 1,-1,2 \rangle} = \vec{n}$

d) $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$, $\vec{r}_0 = \langle 1,1,1 \rangle$

$\langle 1,-1,2 \rangle \cdot \langle x-1, y-1, z-1 \rangle = 0$

$(x-1) - (y-1) + 2(z-1) = 0$

$x - y + 2z - 1 + 1 - 2 = 0$

$\boxed{x - y + 2z = 2}$

c) $\vec{r} = \vec{r}_0 + t\vec{n} = \langle 1,1,1 \rangle + t\langle 1,-1,2 \rangle$

$\boxed{\langle x,y,z \rangle = \langle 1+t, 1-t, 1+2t \rangle}$

x-y plane: $z = 0 = 1 + 2t \rightarrow t = -\frac{1}{2} \rightarrow x = 1 + (-\frac{1}{2}) = \frac{1}{2}$

$y = 1 - (-\frac{1}{2}) = 3/2$ $\boxed{\langle x_0, y_0, z_0 \rangle = \langle \frac{1}{2}, \frac{3}{2}, 0 \rangle}$

e) $P(1,1,1) \rightarrow Q(2,3,3)$: $\vec{PQ} = \langle 2,3,3 \rangle - \langle 1,1,1 \rangle = \langle 1,2,2 \rangle$

$|\vec{PQ}| = \sqrt{1+4+4} = 3$, $\widehat{PQ} = \boxed{\frac{\langle 1,2,2 \rangle}{3} = \hat{u}}$

$D_{\hat{u}} f(1,1,1) = \hat{u} \cdot \vec{\nabla} f(1,1,1)$

$= \frac{\langle 1,2,2 \rangle}{3} \cdot \langle 1,-1,2 \rangle$

$= \frac{1-2+4}{3} = \frac{3}{3} = \boxed{1} > 0$

so f is $\boxed{\text{increasing}}$ in this direction.