MAT2500-03/04 09S Quiz 2 Print Name (Last, First) ______________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P_1(1, 2, 1)$, $P_2(2, 1, 2)$, $P_3(1, 1, 0)$:

a) Write an equation for the plane containing these three points and simplify this equation to its standard form as a linear condition on the coordinates.

b) Evaluate the unit normal vector $\vec{u}$ used in describing this plane.

c) Write scalar parametrized equations ($x = \ldots$, $y = \ldots$, etc) for the line L through the origin $(0, 0, 0)$ which is perpendicular to the plane of the three points.

d) Find the length $c = |\overrightarrow{\mathbf{OP_1}}|$ of $\overrightarrow{\mathbf{OP_1}}$ and scalar component a of $\overrightarrow{\mathbf{OP_1}}$ along $\vec{u}$. [The latter in absolute value is the distance of the plane from the origin.]

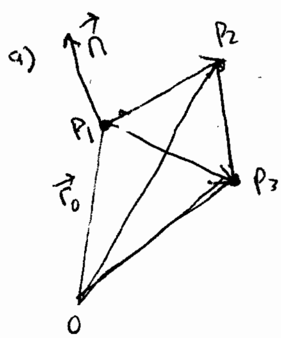
e) Then evaluate the cross product $\vec{b} = \vec{u} \times \overrightarrow{\mathbf{OP_1}}$ showing your hand work.

f) Finally evaluate the length $b = |\vec{b}|$, which should be the magnitude of the projection of $\overrightarrow{\mathbf{OP_1}}$ orthogonal to $\vec{u}$. [This is also the distance between the line L and the point P_1.]

f) Does $a^2 + b^2 = c^2$? Try to draw a rough generic diagram to indicate why this should be true.

▶ solution

a)



$\overrightarrow{P_1P_2} = \langle 2,1,2\rangle - \langle 1,2,1\rangle = \langle 1,-1,1\rangle$

$\overrightarrow{P_1P_3} = \langle 1,1,0\rangle - \langle 1,2,1\rangle = \langle 0,-1,-1\rangle$

$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 0 & -1 & -1 \end{vmatrix} = \langle 1+1, 0+1, -1-0\rangle = \langle 2,1,-1\rangle = \vec{n}$

$\vec{r}_0 = \langle 1,2,1\rangle$

$\vec{n}\cdot(\vec{r}-\vec{r}_0) = 0$

$\to \langle 2,1,-1\rangle \cdot \langle x-1, y-2, z-1\rangle = 0$

$2(x-1) + (y-2) - (z-1) = 0$

$2x + y - z - 2 - 2 + 1 = 0$

$\boxed{2x + y - z = 3}$

b) $\vec{u} = \dfrac{\vec{n}}{|\vec{n}|} = \boxed{\dfrac{\langle 2,1,-1\rangle}{\sqrt{6}}}$

c) $\vec{r} = \langle 0,0,0\rangle + t\vec{n} = t\langle 2,1,-1\rangle = \langle 2t, t, -t\rangle = \langle x,y,z\rangle$

$\boxed{x = 2t,\ y = t,\ z = -t}$ (L is parallel to $\vec{n}$)

d) $c = |\langle 1,2,1\rangle| = \boxed{\sqrt{6}} \approx 2.449$, $a = \vec{u}\cdot\overrightarrow{OP_1} = \dfrac{\langle 2,1,-1\rangle}{\sqrt{6}} \cdot \langle 1,2,1\rangle = \dfrac{2+2-1}{\sqrt{6}} = \boxed{\dfrac{3}{\sqrt{6}} = \dfrac{\sqrt{6}}{2}} \approx 1.225$

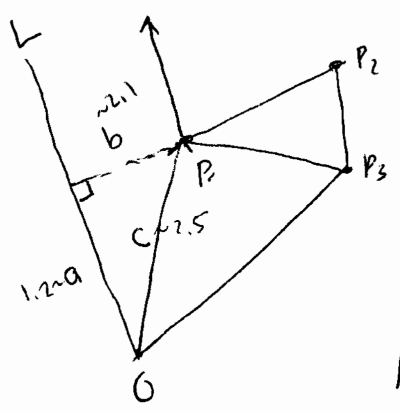
e) $\vec{u} \times \overrightarrow{OP_1} = \dfrac{1}{\sqrt{6}}\langle 2,1,-1\rangle \times \langle 1,2,1\rangle = \dfrac{1}{\sqrt{6}}\begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \dfrac{1}{\sqrt{6}}\langle 1+2, -1-2, 4-1\rangle = \dfrac{1}{\sqrt{6}}\langle 3,-3,3\rangle = \dfrac{3}{\sqrt{6}}\langle 1,-1,1\rangle$

$= \boxed{\dfrac{\sqrt{6}}{2}\langle 1,-1,1\rangle = \vec{b}}$

f) $b = \left|\dfrac{\sqrt{6}}{2}\langle 1,-1,1\rangle\right| = \dfrac{\sqrt{6}}{2}\sqrt{3} = \boxed{3\sqrt{2}}$ ≈

$a^2 + b^2 = \dfrac{6}{4} + \dfrac{18}{4} = 6 = c^2$ ✓

even I cant prevent stupid errors, note corrections, back yourselve up with technology when it counts!



$(a,b,c) = \left(\dfrac{\sqrt{6}}{2}, \sqrt{6}\dfrac{\sqrt{3}}{2}, \sqrt{6}\right)$

$= \sqrt{6}\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}, 1\right)$

30° 60° 90° triangle!

by chance!