MAT2500-03/04 09S Quiz 1 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P1(1,1,1)$, $P2(0,1,2)$, $P3(1,2,0)$:

a) Evaluate the lengths of the three difference vectors $\overrightarrow{\mathbf{P_1P_2}}$, $\overrightarrow{\mathbf{P_1P_3}}$ and $\overrightarrow{\mathbf{P_2P_3}}$ exactly and numerically to 2 decimal place accuracy.

b) Find the angle between $\overrightarrow{\mathbf{P_1P_2}}$ and $\overrightarrow{\mathbf{P_1P_3}}$ exactly in radians (as an inverse trig expression) and numerically in degrees to 1 decimal place accuracy.

c) Knowing the lengths of three sides of the triangle and the one angle between them from parts a) and b), make a rough sketch of this triangle in its own plane, labeling the 3 vertices and three difference vector sides with arrow heads at their terminal points.

d) Find the vector component $\overrightarrow{\mathbf{P_1P_2}}_{\parallel}$ of $\overrightarrow{\mathbf{P_1P_2}}$ along $\overrightarrow{\mathbf{P_1P_3}}$ and the vector component $\overrightarrow{\mathbf{P_1P_2}}_{\perp}$ of $\overrightarrow{\mathbf{P_1P_2}}$ perpendicular to $\overrightarrow{\mathbf{P_1P_3}}$ both exactly and numerically to 2 decimal place accuracy. [Use the dot product to check that your last result is perpendicular to the direction along which we are projecting.]

e) In your sketch of the triangle above, include and label the two projection vectors (arrowheads at their terminal points) and the rectangle associated with the projection/decomposition process you evaluated.

▶ solution

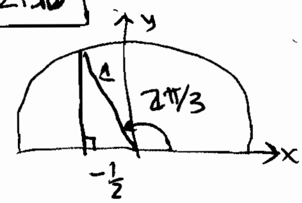
a) $\vec{P_1P_2} = \vec{OP_2} - \vec{OP_1} = \langle 0,1,2\rangle - \langle 1,1,1\rangle = \langle -1,0,1\rangle$ $\quad |\vec{P_1P_2}| = \sqrt{1^2+1^2} = \sqrt{2} \approx 1.41$

$\vec{P_1P_3} = \vec{OP_3} - \vec{OP_1} = \langle 1,2,0\rangle - \langle 1,1,1\rangle = \langle 0,1,-1\rangle$ $\quad |\vec{P_1P_3}| = \sqrt{1^2+1^2} = \sqrt{2} \approx 1.41$

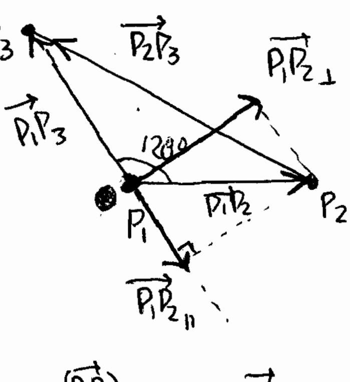
$\vec{P_2P_3} = \vec{OP_3} - \vec{OP_2} = \langle 1,2,0\rangle - \langle 0,1,2\rangle = \langle 1,1,-2\rangle$ $\quad |\vec{P_2P_3}| = \sqrt{1^2+1^2+2^2} = \sqrt{6} \approx 2.45$

b) $\hat{P_1P_2} = \dfrac{\langle -1,0,1\rangle}{\sqrt2}$, $\hat{P_1P_3} = \dfrac{\langle 0,1,-1\rangle}{\sqrt2}$ $\quad \cos\theta = \hat{P_1P_2}\cdot\hat{P_1P_3} = \dfrac{\langle -1,0,1\rangle\cdot\langle 0,1,-1\rangle}{2} = -\dfrac{1}{2}$ ding! →

$\boxed{\theta = \arccos\left(-\tfrac12\right) = \tfrac{2\pi}{3} = 120^\circ}$ (exact)



c)
e)



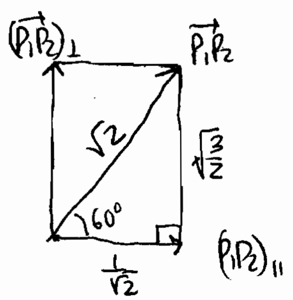
d) $\vec{P_1P_2}\cdot\hat{P_1P_3} = \langle -1,0,1\rangle\cdot\dfrac{\langle 0,1,-1\rangle}{\sqrt2} = -\dfrac{1}{\sqrt2}$

$(\vec{P_1P_2})_{\parallel} = (\vec{P_1P_2}\cdot\hat{P_1P_3})\,\hat{P_1P_3} = -\dfrac{1}{\sqrt2}\dfrac{\langle 0,1,-1\rangle}{\sqrt2} = \boxed{-\tfrac12\langle 0,1,-1\rangle} \boxed{=\langle 0,-0.5,0.5\rangle}$

$(\vec{P_1P_2})_{\perp} = \vec{P_1P_2} - (\vec{P_1P_2})_{\parallel} = \langle -1,0,1\rangle - \langle 0,-\tfrac12,\tfrac12\rangle = \boxed{\langle -1,\tfrac12,\tfrac12\rangle} \boxed{=\langle -1,0.5,0.5\rangle}$

$(\vec{P_1P_2})_{\perp}\cdot\hat{P_1P_3} = \langle -1,\tfrac12,\tfrac12\rangle\cdot\dfrac{\langle 0,1,-1\rangle}{\sqrt2} = \dfrac{1}{2\sqrt2}(1-1) = 0$ ✓

[Bonus:



compute lengths of 3 sides of projection triangle & find sides of 60°-30°-90° triangle with hypotenuse $\sqrt2$]