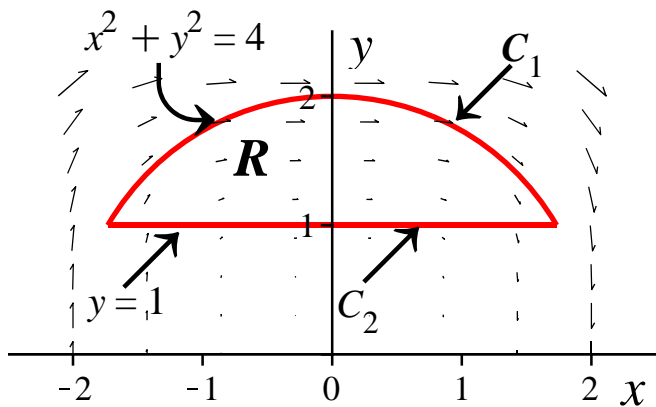


Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. **BOX** final short answers. **LABEL** parts of problem. Keep answers **EXACT** (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Use technology to evaluate any integrals you set up, and give the exact symbolic result together with an appropriate numerical approximation.

1. Given the point $(x, y, z) = (-1, 2, -3)$, find the new coordinates, in each case stating the angles both in radians (exactly, using inverse trig functions) and in degrees (1 decimal place accuracy) and use proper identifying symbols for all coordinates: a) cylindrical coordinates. b) spherical coordinates. Support your work with two diagrams, one of the xy plane and one of the rz half plane, each including a reference triangle locating the point with respect to the axes with all three sides labeled by their lengths and both axes labeled by their coordinate labels. Show clearly how you obtain values of your coordinates from these diagrams. Do the angles look right in these diagrams?



2. a) Describe the region R bounded by the closed counterclockwise directed curve $C = C_1 \cup C_2$ by giving the appropriate intervals of the polar coordinates over the region, and draw in the diagram a typical radial cross-section, labeling its endpoints by the values of the radial coordinate.

b) Use polar coordinates to evaluate $A_y = \iint_R y \, dA$ and

$A = \iint_R 1 \, dA$. What is the value of the average value

$\bar{y} = A_y/A$ of y over the region R ? This ratio is the y coordinate of the centroid of the region. Locate the centroid on the diagram. Does it seem right? Explain.

c) The vector field $\vec{F} = \langle y^3, -x^3 \rangle$ is shown in the diagram. Explain why its line integral around C should be positive or negative.

d) Evaluate the line integral of this vector field directly: $\oint_C \vec{F} \cdot d\vec{r}$. Give the exact value and its decimal approximation to at least several decimal places. Does it have the sign you said it should have in part c)?

e) Check your result by evaluating its equivalent value by Green's theorem: $\iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA$.

f) Set up the same double integral in Cartesian coordinates [support your work with a labeled diagram as usual] and evaluate it as an independent check on your results. Do they agree in value?

3. a) Check that $\vec{F} = \langle 4x - 4y, -4x + 2y \rangle$ satisfies the condition that it admit a potential function, i.e., is a conservative vector field.

b) Find a potential function f for it.

c) Use the potential to evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ over any curve from $(1, 1)$ to $(4, 2)$.

d) Check by evaluating directly the line integral over the straight line segment from the first to the second point. Make a labeled diagram of the line segment to back up your setup.

► solution (on-line)

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in with your answer sheets as a cover page, with the Lastname, FirstName side face up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date: