

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). You are encouraged to use technology to check all of your hand results.

$$1. f(x, y) = x^3 - 6xy + y^2$$

- a) Take it as given that  $(0, 0)$  and  $(6, 18)$  are critical points of this function. Show that the second point satisfies the condition that it is a critical point.  
 b) Use the second derivative test to classify both of these critical points as local minima, local maxima or saddle points. Use words to explain your reasoning.

2. a) Find the equation of the tangent plane to the surface  $z = 3x^2 - y^2 + 2x$  at the point  $(1, -2, 1)$ .

Identify a normal vector  $\vec{n}$  to this plane.

- b) This equation defines a function  $f(x, y)$ . What is the equation of the level curve of  $f$  which passes through the point  $(1, -2)$ ?  
 c) What is the linear approximation function  $L(x, y)$  to  $f$  at this point? Use it to approximate  $f(1.01, -1.98)$ .

3. a) Find the parametrized equations for the normal line to the surface  $x^2 + 2y^2 - 3z^2 = 3$  at the point  $P(2, -1, 1)$ .

b) The left hand side of this equation defines a function  $F(x, y, z)$ . Evaluate the directional derivative of  $F$  in the direction of the origin, using the proper symbol for it.

c) What is the direction (unit vector) in which  $F$  is increasing most rapidly at  $P$  and what is its maximum rate of change at  $P$ ?

d) If  $\langle x, y, z \rangle = \langle t, t^2, t^3 \rangle$  is a parametrized curve, use the chain rule to evaluate the derivative of  $F$  along this curve at  $t = 1$ , namely:  $\left. \frac{d}{dt} F(x(t), y(t), z(t)) \right|_{t=1}$ . (Not very pretty yet. Maybe in Maple 14...)

## ► solution

## ▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: \_\_\_\_\_

Date: \_\_\_\_\_