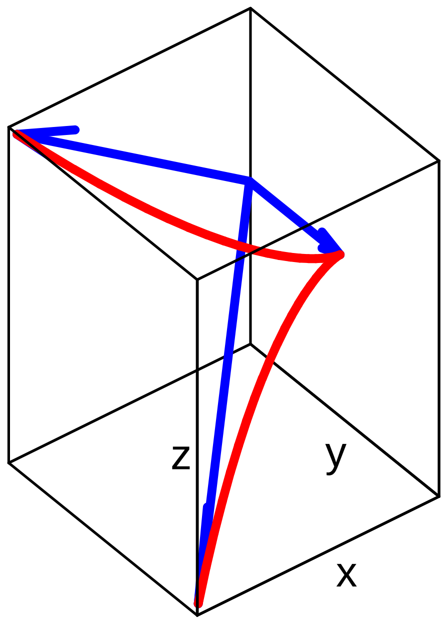
MAT2500-02 08S Test 1 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). You are encouraged to use technology to check all of your hand results.



The parametrized curve $\vec{r}(t) = \langle t^2, 1 - t^3, t^3 \rangle, \ -1 \le t \le 1$ is shown in the figure together with the vectors $\vec{r}(-1), \vec{r}(0), \vec{r}(1)$, all emanating from the origin.

a) Evaluate and simplify $\vec{r}\,'(t), \vec{r}\,''(t), |\vec{r}\,'(t)|, \vec{T}(t)$.

b) Show that this is a plane curve by evaluating the unit vector $\vec{B}(t)$ in the direction of $\vec{r}\,'(t) \times \vec{r}\,''(t)$ to see that it is a constant vector $\vec{B}$.

c) Write the equation of this plane using a simpler normal vector proportional to $\vec{B}$ and the fact that it contains $\vec{r}(0)$.

d) Evaluate the unit normal $\vec{N}(t) = \vec{B} \times \vec{T}(t)$. Confirm that it is orthogonal to the unit tangent.

e) Write down an integral formula for the length of the curve $\vec{r}(t)$ from $t = 0$ to $t = 1$. Factor out the common power of t to see a simple u-substitution that enables the integral to be done easily by hand exactly. Numerically evaluate this exact result and compare numerically to the technology result and to the length of the straight line segment between its endpoints (which is the shortest distance between those points!). How close are the latter numbers?

f) Evaluate the scalar tangential projection $a_T(1)$ along $\vec{T}(1)$ of the acceleration $\vec{a}(1) = \vec{r}\,''(1)$ and its scalar normal projection $a_N(1)$ along $\vec{N}(1)$ exactly and numerically. Does the sum of their squares equal the square of the magnitude of the acceleration as it should?

g) Since $v'(1) = a_T(1)$, is the point on the curve speeding up or slowing down? Why?

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: