MAT2705-02/06 09F Quiz 6 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **[You should use technology for row reductions and determinants.]**

1. $\vec{v}_1 = \langle 2,-2,1\rangle$, $\vec{v}_2 = \langle 3,1,1\rangle$, $\vec{v}_3 = \langle 1,3,0\rangle$, $\vec{v}_4 = \langle 7,-3,3\rangle$,
$\vec{v}_5 = \langle 1,-5,1\rangle$

a) Express v_5 as a linear combination of the remaining 4 vectors, in the most general way. [Final answer: $\vec{v}_5 = \ldots\, \vec{v}_1 + \ldots$]

b) Check that this general linear combination that you find actually evaluates to $\vec{v}_5$.

c) Now express the coefficient vector you found as a linear combination of constant vectors multiplied by arbitrary parameters plus a single constant additive vector.

d) From part c), identify the independent linear relationships among these 4 vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ (i.e., what ***independent*** linear combinations of these vectors equal the zero vector?). Write out these relationships individually.

2. Which of the following sets of vectors are linearly independent (explain your reasoning)?:

a) $\vec{u}_1 = \langle 4,2,3\rangle$, $\vec{u}_2 = \langle 1,-1,1\rangle$, $\vec{u}_3 = \langle 0,1,2\rangle$.

b) $\vec{u}_1 = \langle 1,-2,3\rangle$, $\vec{u}_2 = \langle 2,-3,3\rangle$, $\vec{u}_3 = \langle 3,-4,3\rangle$.

c) $\vec{u}_1 = \langle 2,1,-1,1\rangle$, $\vec{u}_2 = \langle 1,2,1,3\rangle$, $\vec{u}_3 = \langle 4,-3,2,1\rangle$.

▶ solution

1. a) $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 + x_4\vec{v}_4 = \vec{v}_5$

$$\begin{bmatrix} 2 & 3 & 1 & 7 \\ -2 & 1 & 3 & -3 \\ 1 & 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 2 & 3 & 1 & 7 & 1 \\ -2 & 1 & 3 & -3 & -5 \\ 1 & 1 & 0 & 3 & 1 \end{array}\right] \xrightarrow[\text{rref}]{\text{Maple}} \begin{array}{c} \begin{bmatrix} 1 & 0 & -1 & 2 & 2 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ L & L & F & F \end{array} \end{array}$$

$x_3 = t_1,\ x_4 = t_2$

$x_1 = 2 + t_1 - 2t_2$
$x_2 = -1 - t_1 - t_2$
$x_3 = t_1$
$x_4 = t_2$

$$\therefore \boxed{\begin{bmatrix} 1 \\ -5 \\ 1 \end{bmatrix} = (2+t_1-2t_2)\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} + (-1-t_1-t_2)\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} + t_1\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + t_2\begin{bmatrix} 7 \\ -3 \\ 3 \end{bmatrix}}$$

b)
$$= \begin{bmatrix} 4+2t_1-4t_2 & -3-3t_1-3t_2 & +t_1 & +7t_2 \\ -4-2t_1+4t_2 & -1-t_1-t_2 & +3t_1 & -3t_2 \\ 2+t_1-2t_2 & -1-t_1-t_2 & & +3t_2 \end{bmatrix} = \begin{bmatrix} 1+0t_1+0t_2 \\ -5+0t_1+0t_2 \\ 1+0t_1+0t_2 \end{bmatrix} \checkmark$$

c)
$$\vec{x} = \begin{bmatrix} 2+t_1-2t_2 \\ -1-t_1-t_2 \\ t_1 \\ t_2 \end{bmatrix} = t_1\underbrace{\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}}_{\vec{c}_1} + t_2\underbrace{\begin{bmatrix} -2 \\ -1 \\ 0 \\ 4 \end{bmatrix}}_{\vec{c}_2} + \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

1.d)
$$\boxed{\begin{array}{ll} \vec{c}_1: & \vec{v}_1 - \vec{v}_2 + \vec{v}_3 = 0 \\ \vec{c}_2: & -2\vec{v}_1 - \vec{v}_2 + \vec{v}_4 = 0 \end{array}}$$

2 a) $\langle \vec{u}_1|\vec{u}_2|\vec{u}_3\rangle = \begin{bmatrix} 4 & 1 & 0 \\ 2 & -1 & 1 \\ 3 & 1 & 2 \end{bmatrix} = U$

$\det U = -13$ (Maple) $\neq 0$ $\therefore$ linearly independent.

b) $\langle \vec{u}_1|\vec{u}_2|\vec{u}_3\rangle = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 3 & 3 & 3 \end{bmatrix} = U$

$\det U = 0$ (Maple) $\therefore$ linearly dependent

c) $U = \langle \vec{u}_1|\vec{u}_2|\vec{u}_3\rangle = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & -3 \\ -1 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \xrightarrow[\text{Maple}]{\text{rref}} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccc} L & L & L \end{array} \end{array}$

no free variables for system $U\vec{x} = \vec{0}$, no nonzero solns, $\therefore$ linearly independent.

2a) alternate: $U \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ so $U\vec{x} = \vec{0}$ has only $\vec{x} = \vec{0}$ as soln $\rightarrow$ lin. ind.

2b) alternate: $U \xrightarrow{\text{rref}} \begin{array}{c} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccc} L & L & F \end{array} \end{array}$ so $U\vec{x} = \vec{0}$ has nonzero solns, $\rightarrow$ lin dep.