MAT2705-02/06 09F Final Exam Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). You are encouraged to use technology to check all of your hand results.

A two mass, two spring system with parameters $m_1 = 1/2$, $m_2 = 1/12$, $k_1 = 3$, $k_2 = 1/2$ and force per mass $f_2 = 12\cos(t)$ applied to the second mass has the following equations of motion and initial conditions (rest):
$x_1'' = -7x_1 + x_2$, $x_2'' = 6x_1 - 6x_2 + f_2$, $x_1(0) = 0$, $x_2(0) = 0$, $x_1'(0) = 0$, $x_2'(0) = 0$.

a) Solve this system with Maple and write down the solution in vector form. The result is of the form $\vec{x} = \cos(\omega_1 t)\,\vec{X} + \cos(\omega_2 t)\,\vec{Y} + \cos(t)\,\vec{Z}$. Identify the three vectors $\vec{X}, \vec{Y}, \vec{Z}$. Is the response mode a tandem or accordion mode? Identify the two natural modes of the system as the tandem and accordion modes respectively. What are the frequencies and periods of all three modes? Is there a common period T and if so what is its value? What are the $x_1 : x_2$ amplitude ratios for these three modes?

b) Rewrite this system of DEs ***and*** its initial conditions in matrix form for the (column matrix) vector variable $\vec{x} = \langle x_1, x_2 \rangle$, identifying the coefficient matrix A.

c) By hand showing all steps, find the standard eigenvectors $\vec{b}_1, \vec{b}_2$ produced by the solution algorithm with eigenvalues ordered in decreasing order (increasing absolute value). Evaluate the matrix $B = \langle \vec{b}_1 | \vec{b}_2 \rangle$ and its inverse. [Use technology to check your inverse.]

d) What are the slopes M_1, M_2 of the lines through the origin containing the two eigenvectors? On the grid provided, draw in those two lines, labeling them by their corresponding coordinates y_1, y_2 in the positive direction determined by the eigenvectors and then indicate by thicker arrows both eigenvectors, labeled by their symbols. Recall $\vec{x} = B\,\vec{y}$, $\vec{y} = B^{-1}\vec{x}$, where $\vec{y} = \langle y_1, y_2 \rangle$. Also label the x_1, x_2 axes.

e) Now by hand solve the initial value problem stated above before part a).
First write out the new decoupled equations $\vec{y}\,'' = A_D\,\vec{y} + B^{-1}\vec{f}$. Then solve them to find their general solution. Then express the general solution for $\vec{x}$ and impose the initial conditions. Express your final solution in the linear combination form of part a). Does it agree with Maple's solution?

f) For the forcefree system $\vec{f} = \vec{0}$ (set $f_2 = 0$ in the original equations), find the general solution of the DE system by hand using your previous work, expressing the result in vector form as a linear combination of the eigenvectors to show explicitly the two modes of the system.

g) For this forcefree system, solve by hand the initial value problem with initial conditions $\vec{x}(0) = \vec{Z}$, $\vec{x}\,'(0) = \vec{0}$. Check that your solution agrees with Maple. What are the new coordinates (y_1, y_2) of $\vec{Z}$? On your graph, draw in the parallelogram parallel to the new coordinate axes which projects this vector along those axes and identify the sides of the parallelogram on those axes by the number of multiples of the corresponding eigenvector, i.e., $y_1\vec{b}_1$ and $y_2\vec{b}_2$. Do these seem consistent with your plot?

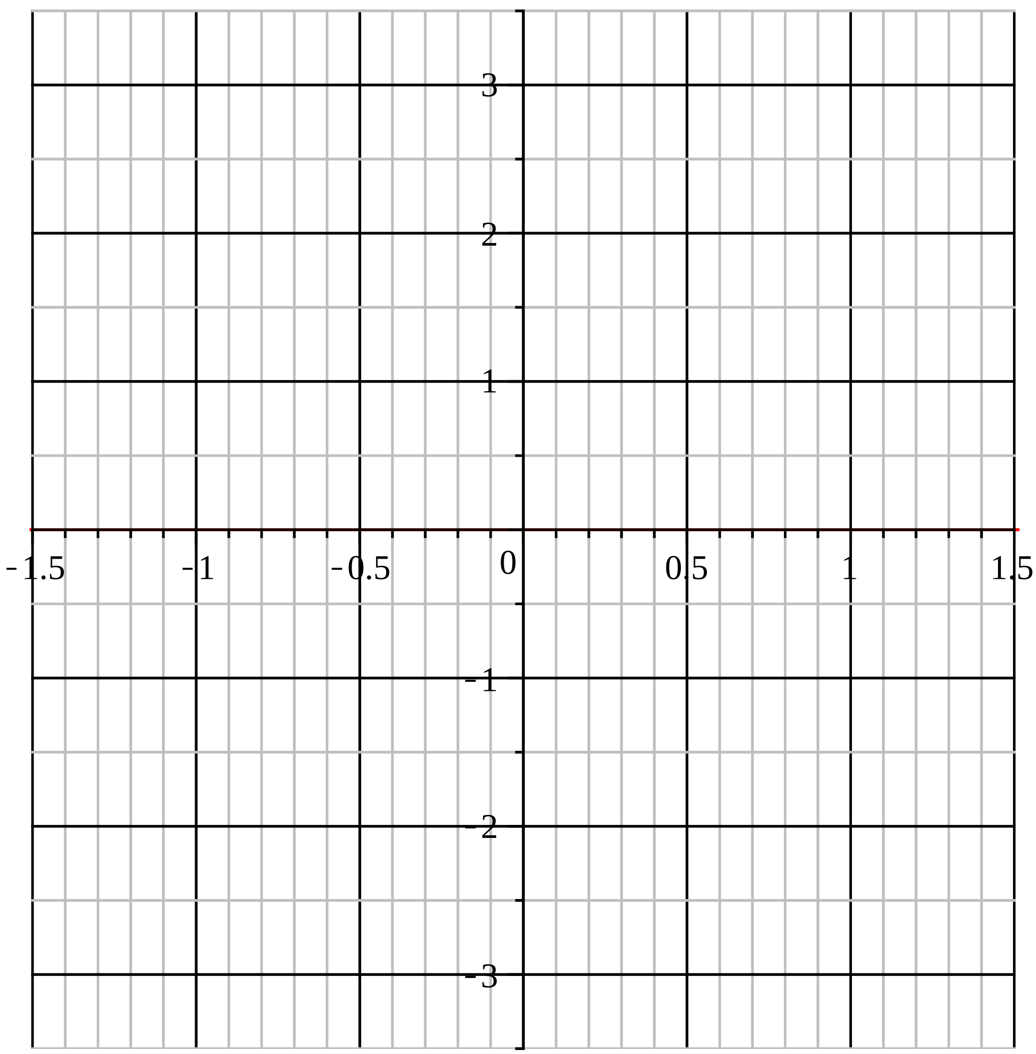
▶ solution

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

gridline default plot window



Be careful drawing in lines with slope $\frac{m}{n}$: n units horizontally, m units vertically.