

MAT2705-02/06 O9F TEST1 Answers

① a) $\frac{dy}{dx} = 2xe^{-y}$ separable

$$\int e^y dy = \int 2x dx$$

$$\ln[e^y = x^2 + C_1]$$

$$y = \ln(x^2 + C) \quad \text{general soln}$$

b) $0 = y(0) = \ln C \rightarrow C = e^0 = 1$

$$y = \ln(x^2 + 1) \quad \text{IVP solution}$$

c) $\frac{dy}{dx} = \frac{1}{x^2+1} \frac{d(x^2+1)}{dx} = \frac{2x}{x^2+1}$

$$\frac{dy}{dx} = 2xe^{-y}$$

$$\frac{2x}{x^2+1} = 2x e^{-\ln(x^2+1)}$$

$$= 2x (x^2+1)^{-1}$$

$$= \frac{2x}{x^2+1} \quad \checkmark$$

② a) $\frac{dy}{dx} = 2ye^{-x}$ linear

$$\frac{dy}{dx} - 2ye^{-x} = 0$$

$$\int -2e^{-x} dx = \frac{2e^{-x}}{e}$$

$$e^{2e^{-x}} \left[\frac{dy}{dx} - 2ye^{-x} = 0 \right]$$

$$\frac{d}{dx}(ye^{2e^{-x}}) = 0$$

$$ye^{2e^{-x}} = C$$

$$y = Ce^{-2e^{-x}}$$

b) $1 = y(0) = Ce^{-2e^0} = Ce^{-2}$

$$C = e^2$$

$$y = e^2 e^{-2e^{-x}} = e^{2-2e^{-x}}$$

c) Agrees! $y(x) = \frac{e^{-2e^{-x}}}{e^{-2}}$
simplifies to our final result.

③ a) $\frac{dT}{dt} = -k(T-350)$ both separable & linear

$$e^{kt} \left[\frac{dT}{dt} + kT = 350k \right]$$

$$\int k dt = e^{kt}$$

$$\frac{d}{dt}(Te^{kt}) = 350ke^{kt}$$

$$e^{-kt} [Te^{kt} = \int 350ke^{kt} dt = 350 \frac{ke^{kt}}{k} + C]$$

$$T = (350e^{kt} + C)e^{-kt}$$

$$= 350 + Ce^{-kt}$$

$40 = T(0) = 350 + C \rightarrow C = -310$

$$T = 350 - 310e^{-kt}$$

$180 = T(4) = 350 - 310e^{-4k}$

$-170 = -310e^{-4k}$

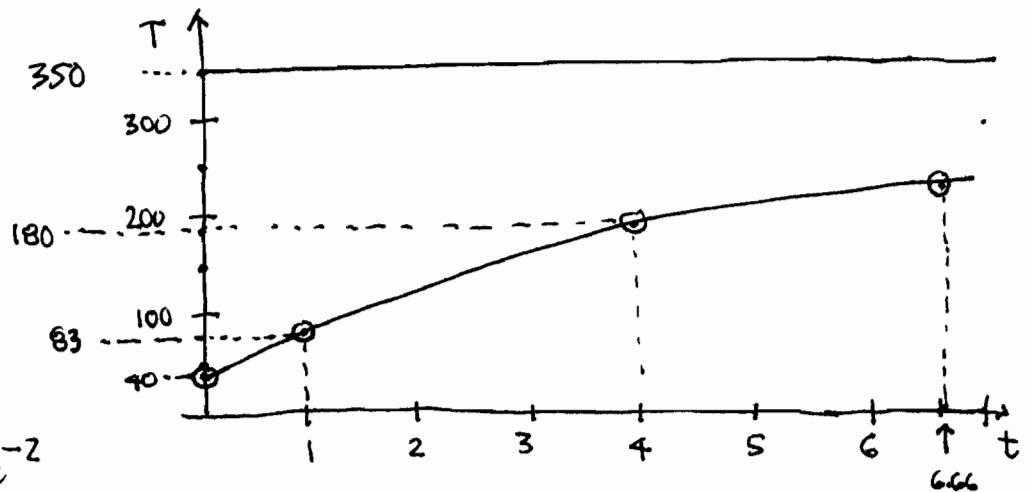
$$e^{4k} = \frac{310}{170} = \frac{31}{17}$$

$4k = \ln(31/17)$

$$k = \frac{1}{4} \ln(31/17) \approx 0.150193$$

$$\tau = 1/k \approx 6.66 \text{ hr}$$

$T(1) = 350 - 310e^{-k} \approx 83.23 \approx 83^\circ$



c) Of course it agrees with Maple, bob always checks all his work to make sure.