

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. a) Evaluate step by step, by hand, the integral $\int_0^1 \int_2^3 12x^2y \, dy \, dx$.

b) Make a diagram shading in the region of integration, and showing a typical cross-section with its directional arrow indicating the inner integration, labeling its endpoints properly. [Label axes, tickmarks, intercepts, etc.]

2) Find the volume under the graph of the function $f(x, y) = x^2 + y^2$ above the unit square $x = 0 \dots 1, y = 0 \dots 1$ in two steps:

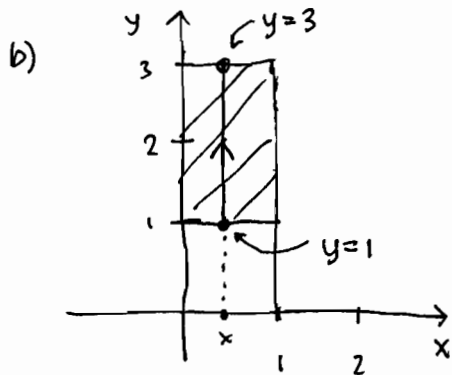
a) First write down an iterated double integral that represents this quantity.

g) Use technology to evaluate it to a number.

► solution

① a)
$$\int_0^1 \int_2^3 12x^2y \, dy \, dx = \int_0^1 30x^2 \, dx = 30\left(\frac{x^3}{3}\right) \Big|_0^1 = 10(1^3 - 0^3) = \boxed{10}$$

$$12x^2\left(\frac{y^2}{2}\right) \Big|_{y=2}^{y=3} = 6x^2(3^2 - 2^2) = 6x^2(9 - 4) = 30x^2$$



limits of integration describe region of integration
 → diagram in plane for double integral

we are converting the limits of integration into the equations of the curves bounding the region of integration.

" y=1 to y=3 as x=0 to x=1 "

② a)
$$\int_0^1 \int_0^1 x^2 + y^2 \, dy \, dx = \int_0^1 \int_0^1 x^2 + y^2 \, dy \, dx$$

either order gives same result

b)
$$\stackrel{\text{(maple)}}{=} \boxed{\frac{2}{3}}$$