MAT2500-02 08S Quiz 2 Print Name (Last, First) ____________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P_1(1, 2, 3)$, $P_2(2, 1, -1)$, $P_3(1, -1, 1)$:

a) Write parametrized equations ($x = \ldots, y = \ldots$, etc) for the line L connecting P_1 to P_2.

b) Write an equation for the plane through the point P_3 which is perpendicular to this line L and simplify this equation to its standard form as a linear condition on the coordinates.

c) Evaluate the unit vector $\vec{u}$ obtained from $\overrightarrow{\mathbf{P_1P_2}}$.

d) Find the length $c = \left|\overrightarrow{\mathbf{P_1P_3}}\right|$ of $\overrightarrow{\mathbf{P_1P_3}}$ and scalar component a of $\overrightarrow{\mathbf{P_1P_3}}$ along $\overrightarrow{\mathbf{P_1P_2}}$.

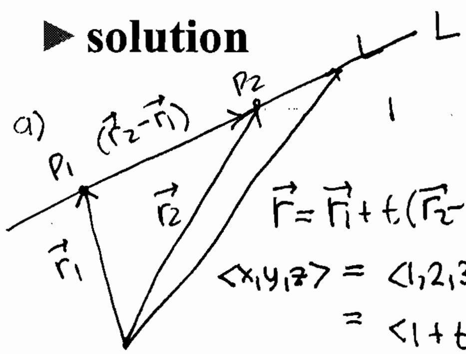
d) Then evaluate the cross product $\vec{b} = \vec{u} \times \overrightarrow{\mathbf{P_1P_3}} = \dfrac{\left(\overrightarrow{\mathbf{P_1P_2}} \times \overrightarrow{\mathbf{P_1P_3}}\right)}{\left|\overrightarrow{\mathbf{P_1P_2}}\right|}$ showing your hand work.

e) Finally evaluate the length $b = \left|\vec{b}\right|$, which should be the magnitude of the projection of $\overrightarrow{\mathbf{P_1P_3}}$ orthogonal to $\overrightarrow{\mathbf{P_1P_2}}$. [This is also the distance between the line L and the point P_3.]

f) Does $a^2 + b^2 = c^2$? Try to draw a rough generic diagram to indicate why this should be true.

▶ solution

a)



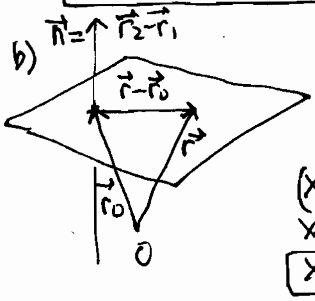
$\vec{r} = \vec{r}_1 + t(\vec{r}_2 - \vec{r}_1)$ $\quad \underbrace{\langle 1,-1,-4\rangle = \vec{r}_2 - \vec{r}_1 = \vec{n}}$

$\langle x,y,z\rangle = \langle 1,2,3\rangle + t(\langle 2,1,-1\rangle - \langle 1,2,3\rangle)$

$= \langle 1+t, 2-t, 3-4t\rangle$

$\boxed{x = 1+t,\ y = 2-t,\ z = 3-4t}$

b)



$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0 \qquad \vec{r}_0 = \overrightarrow{OP_3} = \langle 1,-1,1\rangle$

$\langle 1,-1,-4\rangle \cdot \langle x-1, y-(-1), z-1\rangle = 0$

$(x-1) - (y+1) - 4(z-1) = 0$

$x - y - 4z - 1 - 1 + 4 = 0$

$\boxed{x - y - 4z = -2}$

c) $\overrightarrow{P_1P_2} = \vec{r}_2 - \vec{r}_1 = \langle 1,-1,-4\rangle$, $|\vec{r}_2 - \vec{r}_1| = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

$\hat{u} = \widehat{P_1P_2} = \boxed{\dfrac{\langle 1,-1,-4\rangle}{3\sqrt{2}}}$

d) $\overrightarrow{P_1P_3} = \vec{r}_3 - \vec{r}_1 = \langle 1,-1,1\rangle - \langle 1,2,3\rangle = \langle 0,-3,-2\rangle$

$c = |\overrightarrow{P_1P_3}| = \sqrt{9+4} = \boxed{\sqrt{13}}$

$a = \overrightarrow{P_1P_3} \cdot \hat{u} = \langle 0,-3,-2\rangle \cdot \dfrac{\langle 1,-1,-4\rangle}{3\sqrt{2}} = \dfrac{3+8}{3\sqrt{2}} = \boxed{\dfrac{11}{3\sqrt{2}} = \dfrac{11\sqrt{2}}{6}}$

oops! d2) $\vec{b} = \dfrac{(\vec{r}_2 - \vec{r}_1) \times (\vec{r}_3 - \vec{r}_1)}{3\sqrt{2}}$

$= \dfrac{\langle 1,-1,-4\rangle \times \langle 0,-3,-2\rangle}{3\sqrt{2}}$

$= \dfrac{1}{3\sqrt{2}} \begin{vmatrix} i & j & k \\ 1 & -1 & -4 \\ 0 & -3 & -2 \end{vmatrix} = \dfrac{1}{3\sqrt{2}} \langle 2-12, 0-(-2), -3-0\rangle$

$= \dfrac{1}{3\sqrt{2}} \langle -10, 2, -3\rangle$

e) $|\vec{b}| = \dfrac{1}{3\sqrt{2}} \sqrt{100+4+9} = \boxed{\dfrac{1}{3\sqrt{2}} \sqrt{113}} = b$

f) $a^2 + b^2 = \left(\dfrac{11}{3\sqrt{2}}\right)^2 + \left(\dfrac{\sqrt{113}}{3\sqrt{2}}\right)^2$

$= \dfrac{121 + 113}{18} = \dfrac{234}{18} = \dfrac{117}{9} = \boxed{13 = c^2}$ ✓ yes!