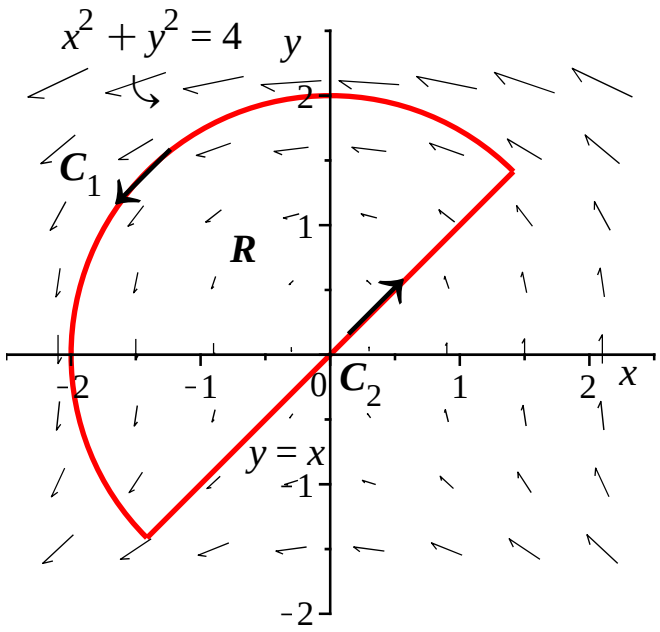
MAT2500-02 08S Final Exam Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).
You may use technology to evaluate any integrals you set up, as long as you can give the exact symbolic result.

1. Given the point $(x, y, z) = (3, 4, -12)$, find the new coordinates, in each case stating the angles both in radians (exactly, using inverse trig functions) and in degrees (1 decimal place accuracy) and use proper identifying symbols for all coordinates: a) cylindrical coordinates. b) spherical coordinates. Support your work with two diagrams, one of the *xy* plane and one of the *rz* half plane, each including a reference triangle locating the point with respect to the axes with all three sides labeled by their lengths and both axes labeled by their coordinate labels. Show clearly how you obtain values of your coordinates from these diagrams.



2. a) Describe the region $\boldsymbol{R}$ bounded by the closed counterclockwise directed curve $\boldsymbol{C} = \boldsymbol{C}_1 \cup \boldsymbol{C}_2$ by giving the appropriate intervals of the polar coordinates over the region, and draw in the diagram a typical radial cross-section, labeling its endpoints by the values of the radial coordinate.

b) Use polar coordinates to evaluate $A_y = \iint_{\boldsymbol{R}} y \, dA$ and $A = \iint_{\boldsymbol{R}} 1 \, dA$. What is the average value of y over the region $\boldsymbol{R}$? The ratio $\overline{y} = A_y / A$ is the y coordinate of the centroid of the region. Does it seem right? Explain.

c) The vector field $\vec{\boldsymbol{F}} = \left\langle -\frac{y^2}{2}, x \right\rangle$ is shown in the diagram. Explain why its line integral around $\boldsymbol{C}$ should be positive or negative.

d) Evaluate the line integral of this vector field directly: $\oint_{\boldsymbol{C}} \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$. Give the exact value and its decimal approximation to at least several decimal places. Does it have the sign you said it should have in part c)?

e) Check your result by evaluating its equivalent value by Green's theorem: $\iint_{\boldsymbol{R}} \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, dA$. Can you see how to use the results of part b) as an additional check on this integral? Explain and do so if you can.

3. a) Evaluate the curl and divergence of the vector fields $\vec{\boldsymbol{F}} = \langle 2xy, x^2 + 2yz, y^2 \rangle$ and $\vec{\boldsymbol{G}} = \langle y e^{-x}, e^{-x}, 2z \rangle$.
b) Which of these is a conservative vector field and why?

4. a) Evaluate the gradient vector field $\vec{\boldsymbol{F}} = \vec{\nabla} f$ associated with the potential function $f = 3x^2 + 2xy + 3y^2$.
b) Use the potential to evaluate the line integral $\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ over any curve from $\left(-\frac{1}{2}, \frac{1}{2}\right)$ to $\left(\frac{1}{2}, \frac{1}{2}\right)$.

solution (on-line)

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in with your answer sheets as a cover page, with the Lastname, FirstName side face up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: