

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

For each hand integration step, state the antiderivative formula used before substituting limits into it:

$$\int_a^b f(x) dx = F(x)|_{x=a}^{x=b} = F(b) - F(a) .$$

1. $\int_0^4 \int_{\frac{y}{2}}^{\sqrt{y}} e^{3x^2 - x^3} dx dy$ a) Evaluate this numerically to at least 6 digit accuracy using technology.

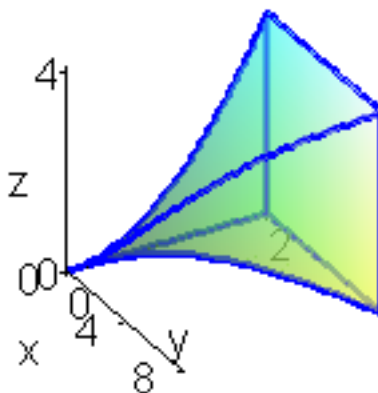
- b) Make a completely labeled diagram of the region of integration with a typical labeled cross-section representing the current iteration of the integral.
- c) Make a new completely labeled diagram corresponding to the reversed order of integration.
- d) State the new integral with the order of integration reversed.
- e) Evaluate the new integral by hand step by step exactly and give a numerical value for this exact result.
- f) Do you get the same result as in part a)?

2. $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} dy dx$ a) Make a completely labeled diagram of the region of integration with a typical labeled cross-section representing the current iteration of the integral.

- b) Make a new such diagram corresponding to polar coordinate integration.
- c) Express the integral in polar coordinates.
- d) Evaluate the integral exactly. Does your result agree with the numerical value of the original integral?

3. $\int_0^2 \int_0^{y^3} \int_0^{y^2} f(x, y, z) dz dx dy$

- a) Evaluate this integral by hand step by step first for $f(x, y, z) = z$ and then for $f(x, y, z) = 1$ and then evaluate their ratio to get the z-component of the centroid.
- b) Use technology to evaluate these integrals. Do your results agree? If not, can you find your error?
- c) Rewrite these integrals a equivalent integrals in the order $\iiint \dots dx dy dz$, justifying your limits of integration with completely labeled diagrams and calculations.
- d) Evaluate your new integrals step by step and by using technology. Do they agree with your previous results?



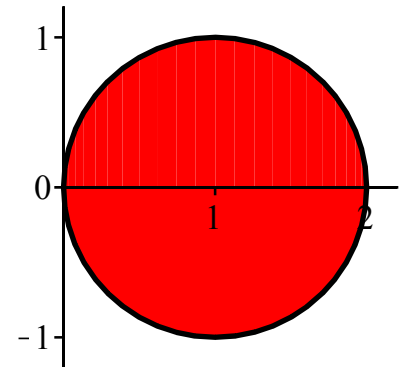
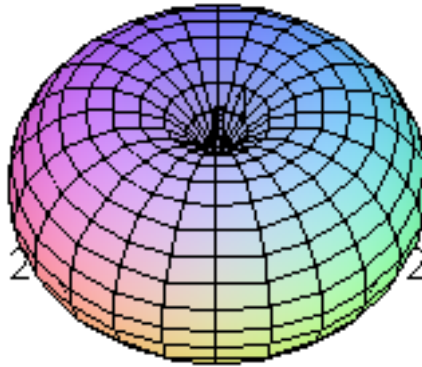
4. Consider the pinched torus described in cylindrical coordinates by $(r - 1)^2 + z^2 = 1$, which encloses a solid region R of space shown in the figure.

a) Express the equation for this surface in spherical coordinates.

c) Express the triple integral

$$\iiint_R z^2 dV$$

in cylindrical coordinates, and evaluate step by step.



b) Make your own fully labeled r - z diagram of the solid first showing a typical vertical cross-section with its directional arrow indicating the inner integration of a triple integral over R in cylindrical coordinates, labeling its endpoints properly, and then repeat for the spherical coordinate iteration.

d) Express the triple integrals $Vz^2 = \iiint_R z^2 dV$ and $V = \iiint_R 1 dV$ in spherical coordinates, and evaluate each

step by step.

e) Using your results for part d), show that the average value of z^2 over the torus is $1/4$.

► solution (on-line)

No collaboration. You may only talk to bob. See test rules [on-line](#). Print out and attach hand labeled Maple supporting documentation if you wish to include it with your work.

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies and hand in with your answer sheets as a cover page, with the Lastname, FirstName side face up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date: