

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. **BOX** final short answers. **LABEL** parts of problem. Keep answers **EXACT** (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology to find the roots of polynomials. [No mercy for mistakes here.]**

1. $y'' + 4y' + 8y = \cos(\omega t)$, $\omega \geq 0$.

a) Use the method of undetermined coefficients to find the steady state sinusoidal solution (the particular solution y_p) of this driven harmonic oscillator DE.

b) Evaluate the amplitude $A(\omega)$ of this sinusoidal function. Show that it simplifies to the formula

$$A(\omega) = (64 + \omega^4)^{-\frac{1}{2}}.$$

c) Use calculus to show that the peak amplitude occurs at the value $\omega = 0$. Evaluate $A(0)$ exactly and numerically.

[This system has the value of the quality factor $Q = 1/\sqrt{2} \approx 0.707$, which is the lowest value at which there is a horizontal tangent on the amplitude plot for some nonnegative value of the frequency.]

d) Use technology to plot $A(\omega)$ for $\omega = 0 \dots 20$. Sketch what you see, labeling the axes, tickmarks and the vertical intercept.

e) Try to numerically solve for the value of the frequency ω for which $A(\omega) = \frac{1}{2} A(0)$, i.e., where the response amplitude is half the response at zero frequency.

► solution