MAT2705-01 08F Quiz 6 Print Name (Last, First) ____________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $y''+3y'+2y=0,\ y(0)=0,\ y'(0)=1$.

a) Verify that the general solution $y(x)=C1\, e^{-x}+C2\, e^{-2x}$ actually solves the DE.

b) Using matrix methods find the solution which satisfies the initial conditions, showing all work.

c) Use technology to plot your result for $x=0..5$ and make a rough sketch of what you see, labeling the axes with variable names and key tickmarks on your sketch.

d) Use calculus to determine exactly by hand (rules of exponents and logs!) the x and y values of the obvious maximum point on the graph and then their approximate values, but if you get stuck on solving the derivative condition exactly, use technology to find the approximate values to 4 decimal places in any way you can. Do the numbers you found agree with what your eyes see in the technology plot? [Yes or no, with an explanation would be a good response.]

▶ solution

1 a)
$$\left.\begin{aligned} y &= c_1 e^{-x}+c_2 e^{-2x} \\ y' &= -c_1 e^{-x}-2c_2 e^{-2x} \\ y'' &= c_1 e^{-x}+4c_2 e^{-2x} \end{aligned}\right\}$$

$$\begin{aligned} y''+3y'+2y = {} & c_1 e^{-x}+4c_2 e^{-2x} \\ & +3(-c_1 e^{-x}-2c_2 e^{-2x}) \\ & +2(c_1 e^{-x}+c_2 e^{-2x}) \end{aligned} = \overbrace{(1-3+2)}^{0} c_1 e^{-x} + \underbrace{(4-6+2)}_{0} c_2 e^{-2x} = 0 \checkmark$$

$$y(0) = c_1 + c_2 = 0$$

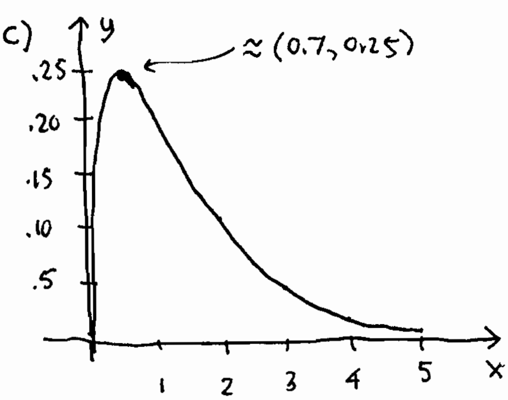
b) $$y'(0) = -c_1 - 2c_2 = 1$$

matrix form: $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{\text{soln}} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \dfrac{1}{-2+1}\begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\therefore \boxed{y = e^{-x} - e^{-2x}}$$

c)



d)
$$y' = -e^{-x}+2e^{-2x} = 0$$
$$e^{2x}(e^{-x} = 2e^{-2x})$$
$$e^{x} = 2$$
$$x = \ln 2 \approx 0.6931$$
$$\begin{aligned} y(\ln 2) &= e^{-\ln 2} - e^{-2\ln 2} \\ &= (e^{\ln 2})^{-1} - (e^{\ln 2})^{-2} \\ &= 2^{-1} - 2^{-2} = \tfrac{1}{2} - \tfrac{1}{4} = \tfrac{1}{4} = 0.25 \end{aligned}$$

$$\boxed{\text{max: } (x,y) = (\ln 2, 1/4) \approx (.6931, 0.25)}$$

yes, excellent agreement with rough estimate given in figure.