MAT2705-01 08F Quiz 4 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

$4x_1 + 2x_2 + 2x_3 + 2x_4 = 0$
$3x_1 + x_2 + 3x_3 + 3x_4 = 0$

a) Write down the coefficient matrix **A**, the RHS matrix **b** and the augmented matrix **C** = <**A** | **b**> for this linear system of equations.
b) With technology (identify your choice!), reduce this matrix to its ReducedRowEchelonForm.
c) Write out the pair of equations that correspond to the reduced matrix. Identify the leading variables and the free variables (don't forget!) and solve. State your solution in the scalar form: $x_1 = ..., x_2 = ...,$ etc.
d) Now state your solution in column matrix ("vector") form $\boldsymbol{x} = ...$ and evaluate by hand the matrix product $\boldsymbol{A}\,\boldsymbol{x}$. Does it equal **0** as it should?
e) Now re-express the solution as an arbitrary linear combination of fixed vectors by factoring out the vector of coefficients of each of the free parameters in the solution.

▶ solution

a) $A = \begin{bmatrix} 4 & 2 & 2 & 2 \\ 3 & 1 & 3 & 3 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 2 & 2 & 2 \\ 3 & 1 & 3 & 3 \end{bmatrix}$

b) $C \xrightarrow{\text{Maple}} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \\ \hline \end{array}\begin{bmatrix} 1 & 0 & 2 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$
$\quad\quad$ L L F F

c) $x_1 + 2x_3 + x_4 = 0$
$\quad x_2 - 3x_3 = 0$

Leading $\quad$ Free: $x_3 = t_1$ (or t)
$\quad\quad\quad\quad\quad\quad x_4 = t_2$ (or s)

$\rightarrow \begin{aligned} x_1 + 2t_1 + t_2 &= 0 \\ x_2 - 3t_1 &= 0 \end{aligned} \rightarrow \begin{aligned} x_1 &= -2t_1 - t_2 \\ x_3 &= 3t_1 \end{aligned}$

soln: $\boxed{x_1 = -2t_1 - t_2,\ x_2 = 3t_1,\ x_3 = t_1,\ x_4 = t_2}$

d) $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2t_1 - t_2 \\ 3t_1 \\ t_1 \\ t_2 \end{bmatrix}$ $\overset{e)}{=}$ $t_1 \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

this is linear in (t_1, t_2) but the coefficients are vectors!

$\downarrow$

$A\vec{x} = \begin{bmatrix} 4 & 2 & 2 & 4 \\ 3 & 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} -2t_1 - t_2 \\ -3t_1 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 4(-2t_1 - t_2) + 2(-3t_1) + 2(t_1) + 4(t_2) \\ 3(-2t_1 - t_2) + 1(3t_1) + 3(t_1) + 3(t_2) \end{bmatrix}$

$= \begin{bmatrix} \underbrace{(-8+6+2)}_{=0} t_1 + \underbrace{(-4+4)}_{=0} t_2 \\ \underbrace{(-6+3+3)}_{=0} t_1 + \underbrace{(-3+3)}_{=0} t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ✓ yes!