

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

$$2x_1 + 4x_2 + 6x_3 = 8$$

$$3x_1 + 5x_2 + 7x_3 = 9$$

a) Write down the coefficient matrix A , the RHS matrix b and the augmented matrix $C = \langle A|b \rangle$ for this linear system of equations.

b) Reduce this matrix to its *ReducedRowEchelonForm* in a few easy steps, annotating the *MultiplyRow*, *AddRow*, *SwapRow* operations you apply to each successive matrix in the process, using your own words or the notation:

$$R_1 \rightarrow 3R_1, R_1 \rightarrow R_1 + 2R_2, R_1 \leftrightarrow R_2.$$

c) Write out the pair of equations that correspond to the reduced matrix. Identify the leading variables ("L") and the free variables ("F") and solve for the leading variables, assigning arbitrary parameters to the free variables. State your solution in the form:

$$x_1 = \dots, x_2 = \dots, x_3 = \dots$$

d) Check by backsubstitution of your solution into the original system that it is satisfied.

► solution

a) $A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} \quad b = \begin{bmatrix} 8 \\ 9 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \end{bmatrix} \xrightarrow{\substack{R_1 \rightarrow \frac{1}{2}R_1 \\ R_2 \rightarrow R_2 - 3R_1}} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 5 & 7 & 9 \end{bmatrix} \xrightarrow{R_2 \rightarrow -R_2} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & -1 & -2 & -3 \end{bmatrix}$
-3 -6 -9 -12 L L F

c)
$$\begin{array}{l} x_1 - x_3 = -2 \\ x_2 + 2x_3 = 3 \end{array} \quad \begin{array}{l} x_1 - t = -2 \rightarrow \\ x_2 + 2t = 3 \rightarrow \end{array} \quad \boxed{\begin{array}{l} x_1 = t - 2 \\ x_2 = -2t + 3 \\ x_3 = t \end{array}}$$

↑ ↑ ↑
L L F
 $x_3 = t$ →

d) $2(t-2) + 4(-2t+3) + 6(t) \stackrel{?}{=} 8$
 $\llcorner \underbrace{(2-8+6)}_0 t + \underbrace{(-4+12)}_8 = 8 \checkmark$

$3(t-2) + 5(-2t+3) + 7(t) \stackrel{?}{=} 9$
 $\llcorner \underbrace{(3-10+7)}_0 t + \underbrace{(-6+15)}_9 = 9 \checkmark$