MAT2705-01 08F Final Exam Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). **You may use technology for row reductions, determinants, inverses and root finding.** You are encouraged to use technology to check all of your hand results. ***You should use technology row reduction or matrix inverses to solve 2x2 matrix equations to avoid fractional arithmetic errors which make the numbers much more complicated. Ditto for quadratic equations.***

1. A coupled system of ODEs has the following equations of motion
$x_1' = -4\,x_1 + 8\,x_2,\ x_2' = x_1 - 6\,x_2,\ x_1(0) = 0,\ x_2(0) = 3$.
a) Rewrite this system of DEs and its initial conditions in matrix form for the vector variable $\vec{\boldsymbol{x}} = \langle x_1, x_2 \rangle$, identifying the coefficient matrix *A*.
b) By hand, showing all steps, find the standard eigenvectors $\vec{\boldsymbol{b}}_1, \vec{\boldsymbol{b}}_2$ produced by the solution algorithm with eigenvalues ordered in decreasing order. Evaluate the matrix $B = \langle \vec{\boldsymbol{b}}_1 | \vec{\boldsymbol{b}}_2 \rangle$
and its inverse.
c) What are the equations $x_2 = m\,x_1$ in the x_1-x_2 plane of the two eigenspaces (lines!) associated with these two eigenvectors?
d) On the grid provided, indicate by thick arrows both eigenvectors, extending them to labeled coordinate y_1, y_2 axes, and draw in the vector $\vec{\boldsymbol{x}}(0)$ and its parallelogram projection onto those axes. Evaluate the new coordinates of $\vec{\boldsymbol{x}}(0)$ using matrix methods. Do their values seem consistent with your drawing? Explain.
e) Find the general solution of the DE system by hand, showing all steps.
f) Find the solution which satisfies the initial conditions, by hand, showing all steps. Express it first in vector form as an explicit linear combination of the two eigenvectors, and then in scalar form giving expressions for each of the two scalar variables.
g) What are the two characteristic times associated with the exponential behavior exhibited in this problem. Sketch the graphs of the two variables of part f) on the same axes for 5 times the longest such characteristic time.

2. A coupled system of ODEs has the following equations of motion
$\vec{\boldsymbol{x}}'' = A\,\vec{\boldsymbol{x}},\ \vec{\boldsymbol{x}}(0) = \langle 0, 3 \rangle,\ \vec{\boldsymbol{x}}'(0) = \langle 0, 0 \rangle$, where *A* is the same matrix as in problem 1.
a) Express the DEs and initial conditions in scalar form.
b) Write out the DEs for the new variables $\vec{\boldsymbol{y}} = B^{-1}\,\vec{\boldsymbol{x}}$ first in matrix form and then in scalar form for the individual variables y_1, y_2. Find the general solution and express $\vec{\boldsymbol{x}}$ in terms of it.
c) Find the solution of the original DEs satisfying the initial conditions and express it first in vector form as an explicit linear combination of the two eigenvectors, and then in scalar form giving expressions for the individual variables.
d) This has two modes: a "tandem" mode (same sign values of the two unknowns) and an "accordian" mode (opposite sign values of the two unknowns). What are the frequencies and periods of these two oscillating modes respectively (indicate which is which)? What is the common period of the periodic free motion of this system? Sketch the clearly labeled individual variables x_1, x_2 versus *t* for exactly one period of this motion.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

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