MAT2705-01 08F Test 2 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses.**

1. a) Express $\langle 5, 5\rangle$ as a linear combination of $\langle 2, -4\rangle$ and $\langle 1, 3\rangle$. Use matrix techniques to state the necessary system of equations and use the matrix inverse to solve that system.
b) Check that this linear combination evaluates to the original vector.
c) Make a rough sketch of the 3 vectors and the parallelogram which helps us interpret the linear combination, i.e., which illustrates how we graphically represent $\langle 5, 5\rangle$ in terms of the first two vectors.

2. a) For $A = \langle \vec{v}_1 | \vec{v}_2 | \vec{v}_3 | \vec{v}_4 \rangle$ with $\vec{v}_1 = \langle 1, -1, 1\rangle$, $\vec{v}_2 = \langle 1, 1, 1\rangle$, $\vec{v}_3 = \langle 1, -2, 4\rangle$, $\vec{v}_4 = \langle 1, 2, 0\rangle$ and $\vec{x} = \langle x_1, x_2, x_3, x_4\rangle$, $\vec{0} = \langle 0, 0, 0\rangle$, solve the equations $A\,\vec{x} = \vec{0}$, first stating the augmented matrix and its RREF form, the corresponding reduced equations, identifying free and leading variables.
b) From your general solution write down a basis $\{\vec{u}_1, \ldots\}$ of the solution space.
c) Use your work to write down the independent linear relationships which exist among the four vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$. Check that each such linear combination equals the zero vector.

3. $A_1 = \begin{bmatrix} 2 & -3 & -3 \\ -1 & 1 & 2 \\ 3 & -5 & -4 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $A_3 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$. Which of these triplets of column matrices, each arranged in a square matrix, consists of linearly independent vectors? Explain why.

solution

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: