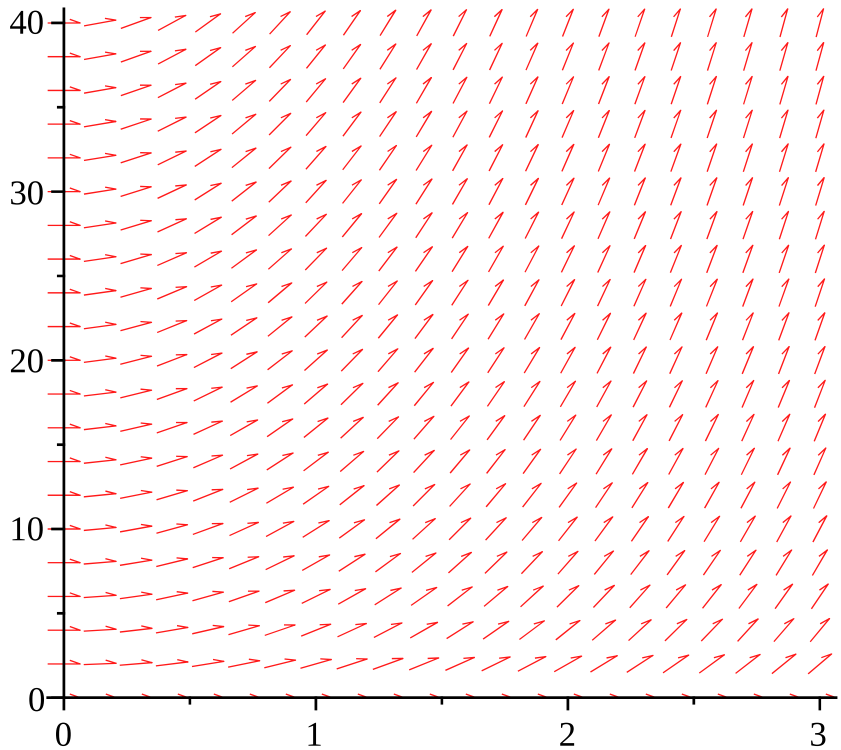
MAT2705-01 08F Test 1 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC): only for antiderivatives, finding a root, or checking.



1. $y' = 3\,x\,y^{\frac{1}{2}}, \quad y(0) = 9$

a) Find the general solution of this differential equation by hand.
b) Find the solution which satisfies the initial condition.
c) Check by backsubstitution that your solution of this initial value problem satisfies the differential equation.
d) At what value of $x > 0$ will y reach the value 36?
e) Use the direction field at the left to illustrate this initial value problem and its solution, and to confirm that your result for d) makes sense graphically. Explain your response and annotate the graph showing your reasoning and estimation.

2. If we measure the amount of pollutant in water by its volume rather than its weight or mass, then the concentration (amount/volume) becomes a dimensionless fraction and so can be measured as a percentage of volume.
Consider a reservoir with a volume of $V_0 = 8000$ million ft^3 and an initial pollution concentration of 0.25%: $c(0) = \frac{x(0)}{V_0} = .0025$, where $x(t)$ describes the amount (volume in million ft^3) of pollutant in the water at time t in days from the initial time. There is a daily inflow of $r_i = 500$ million ft^3 with a pollution concentration of 0.05%: $c_i = 0.0005$ and an equal daily outflow $r_o = r_i$ of the well-mixed water in the reservoir. How long will it take to reduce the pollutant concentration in the reservoir to 0.10%?
[*Don't panic, continue reading.*]

a) Since $\frac{dx}{dt} = r_i\,c_i - \frac{r_o}{V_0}\,x$, $r_i\,c_i = \frac{1}{4}$, $\frac{r_o}{V_0} = \frac{1}{16}$ and $c(0)\,V_0 = 20$, the word problem translates into the following initial value problem:
$\frac{dx}{dt} = \frac{1}{4} - \frac{x}{16}$, $x(0) = 20$. Solve this by hand.
b) After a long enough time what constant value does the pollutant amount $x(t)$ approach?
c) Recalling that the pollutant concentration in the lake is $c(t) = \frac{x(t)}{V_0}$, solve the mathematical condition required to answer the word problem exactly using rules of exponentials and logarithms, then approximate and state the answer with an appropriate number of significant digits with units as an English sentence.
d) Aren't you glad I translated the word problem for you? Who needs such stress on an in-class exam? [rhetorical questions, :-)]

(turn over please)

Bonus question.
If you are completely confident that everything you have done above is correct and you have checked all of your answers, consider the following:
2e) What is the characteristic time for this exponential decay process? How long instead does it take for the relevant exponential to decay by a factor of 2, i.e., what is the half-life of this decay process? How many half-lives does the answer for part c) equal? Draw a diagram illustrating the initial value problem solution and these decay times.

▶ solution

▼ pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: