MAT2705-01/04 07S Quiz 5 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

[You may use technology for row reductions, backsubstitutions and determinants although these may easily be done by hand.]

1. $v_1 = \langle 1, 1, 1, 1\rangle$, $v_2 = \langle 1, 0, 1, 0\rangle$, $v_3 = \langle 2, 1, 2, 1\rangle$, $v_4 = \langle 2, 3, 2, 3\rangle$, $v_5 = \langle 1, 2, 1, 2\rangle$.

a) Express v_5 as a linear combination of the remaining 4 vectors, in the most general way. [Final answer: $v_5 = \ldots$]

b) Check that this general linear combination that you find actually evaluates to v_5.

c) Find the independent linear relationships among these 4 vectors. Write out these relationships individually.

OPTIONAL. If you have time while others are finishing up, try this:

2. Demonstrate that the following vectors are linearly independent:
$u_1 = \langle 1, 1, 4, 4\rangle$, $u_2 = \langle 2, 2, 3, 3\rangle$, $u_3 = \langle 3, -3, 2, -2\rangle$, $u_4 = \langle 4, -4, 1, -1\rangle$.

▶ solution

① a) $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 + x_4\vec{v}_3 = \vec{v}_5$

$$\begin{bmatrix}1&1&2&2\\1&0&1&3\\1&1&2&2\\1&0&1&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}1\\2\\1\\2\end{bmatrix} \to \begin{bmatrix}1&1&2&2&1\\1&0&1&3&2\\1&1&2&2&1\\1&0&1&3&2\end{bmatrix} \xrightarrow[\text{maple}]{\text{rref}} \begin{array}{ccccc} L & L & F & F & \\ x_1 & x_2 & x_3 & x_4 & \end{array} \begin{bmatrix}1&0&1&3&2\\0&1&1&-1&-1\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$$

$x_1 + x_3 + 3x_4 = 2$
$x_2 + x_3 - x_4 = -1$
$0 = 0$
$0 = 0$

$x_3 = t_1$ $\quad x_1 = 2 - t_1 - 3t_2$
$x_4 = t_2$ $\quad x_2 = -1 - t_1 + t_2$

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}2-t_1-3t_2\\-1-t_1+t_2\\t_1\\t_2\end{bmatrix}$$

so: $\boxed{(2-t_1-3t_2)\vec{v}_1 + (-1-t_1+t_2)\vec{v}_2 + t_1\vec{v}_3 + t_2\vec{v}_4 = \vec{v}_5}$

b) $$(2-t_1-3t_2)\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + (-1-t_1+t_2)\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + t_1\begin{bmatrix}2\\1\\2\\1\end{bmatrix} + t_2\begin{bmatrix}2\\3\\2\\3\end{bmatrix} = \begin{bmatrix}(2-t_1-3t_2)+(-1-t_1+t_2)+2t_1+2t_2\\(2-t_1-3t_2)+t_1+3t_2\\(2-t_1-3t_2)+(-1-t_1+t_2)+2t_1+2t_2\\(2-t_1-3t_2)+t_1+3t_2\end{bmatrix}$$

$$= \begin{bmatrix}1+t_1(-1-1+2)+t_2(-3+1+2)\\2+t_1(-1+1)\\1+t_1(-1-1+2)+t_2(-3+1+2)\\2+t_1(-1+1)+t_2(-3+3)\end{bmatrix} = \begin{bmatrix}1\\2\\1\\2\end{bmatrix} \checkmark$$

c) replacing $\vec{v}_5$ by $\vec{0}$ leads to $\vec{x} = t_1\begin{bmatrix}-1\\-1\\1\\0\end{bmatrix} + t_2\begin{bmatrix}-3\\1\\0\\1\end{bmatrix}$ general soln

2 independent solns.

so $\boxed{\begin{aligned}-\vec{v}_1-\vec{v}_2+\vec{v}_3 &= 0\\ -3\vec{v}_1+\vec{v}_2+\vec{v}_4 &= 0\end{aligned}}$

② $A = \langle \vec{u}_1|\vec{u}_2|\vec{u}_3|\vec{u}_4\rangle = \begin{bmatrix}1&2&3&4\\1&2&-3&-4\\4&3&2&1\\4&3&-2&-1\end{bmatrix}$.

$|A| \overset{\text{maple}}{=} -100 \neq 0$ therefore A^{-1} exists

$A\vec{x} = \vec{0} \to \vec{x} = A^{-1}\vec{0} = \vec{0}$

no nontrivial linear combinations of the vectors equal the zero vector so they are linearly independent.

(enough to find $|A| \neq 0$ for lin. ind.)