

MAT2705-01/04 075 TEST 1 ANSWERS

① a) $\frac{dy}{dx} = -2xy$ separable

$\int y^{-1} dy = \int -2x dx$

$|\ln|y|| = -x^2 + C_1$

$|y| = e^{-x^2 + C_1} = e^{C_1} e^{-x^2}$

$y = \pm e^{C_1} e^{-x^2}$

$y = C e^{-x^2}$ general soln

b) $10 = y(0) = C e^{-0^2} = C$

$\therefore y = 10 e^{-x^2}$

c) $100 e^{-x^2} = (0.01)(100)$

$e^{-x^2} = .01$

$\ln[e^{x^2} = 100]$

$x^2 = \ln 100 = \ln 10^2 = 2 \ln 10$

$x = \pm \sqrt{\ln 100} = \pm \sqrt{2 \ln 10}$

but $x > 0$: $x = \sqrt{\ln 100} = \sqrt{2 \ln 10} \approx 2.146$

This is actually a 1 tank mixing problem for $t \geq 0$:

② b) $\frac{dx}{dt} + \frac{3}{1+2t} x = 5$ linear, in standard form

$\int \frac{3}{1+2t} dt = \frac{3}{2} \ln|1+2t| \rightarrow$ absolute unnecessary here
 $= (1+2t)^{3/2}$

$(1+2t)^{3/2} \left[\frac{dx}{dt} + \frac{3}{1+2t} x \right] = 5 (1+2t)^{3/2}$

$\frac{d}{dt} (x(1+2t)^{3/2}) = 5 (1+2t)^{3/2}$

$x(1+2t)^{3/2} = \int 5(1+2t)^{3/2} dt$
 $= \frac{5}{2} \frac{(1+2t)^{5/2}}{5/2} + C = (1+2t)^{5/2} + C$

$x = \frac{(1+2t)^{5/2} + C}{(1+2t)^{3/2}} = 1+2t + C(1+2t)^{-3/2}$
 If $1+2t > 0 \dots$ general soln

② c) $\frac{1}{2} = x(t) = 1+2t + C(1+2t)^{-3/2} = 1+C$
 $C = \frac{1}{2} - 1 = -\frac{1}{2}$

$x = 1+2t - \frac{1}{2(1+2t)^{3/2}}$

d) $\frac{dx}{dt} = 0+2 - \frac{1}{2}(-\frac{3}{2})(1+2t)^{-5/2} (0+2)$

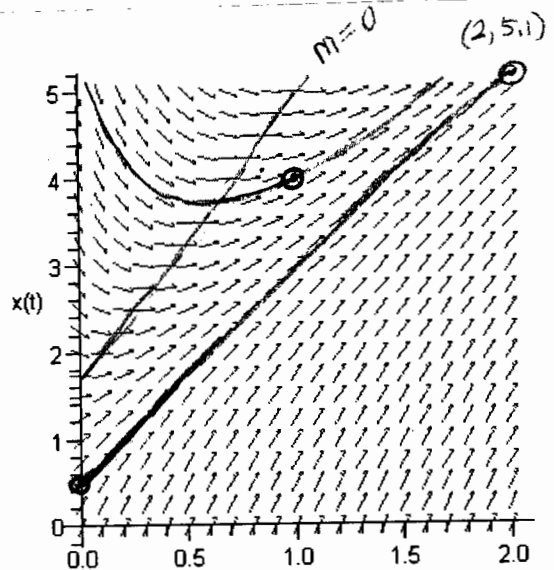
$= 2 + \frac{3}{2}(1+2t)^{-5/2}$

$\frac{dx}{dt} + \frac{3}{1+2t} x = 5$

$2 + \frac{3}{2}(1+2t)^{-5/2} + \frac{3}{1+2t} \left[1+2t - \frac{1}{2(1+2t)^{3/2}} \right] \stackrel{?}{=} 5$
 $3 - \frac{3}{2}(1+2t)^{-3/2-1} = -5/2$

$2 + 3 = 5$
 $5 = 5 \checkmark$

a) f) $x(0) = \frac{1}{2} \Leftrightarrow t=0, x=\frac{1}{2} \Leftrightarrow$ pt: $(0, \frac{1}{2})$
 $x(1) = 4 \Leftrightarrow t=1, x=4 \Leftrightarrow$ pt: $(1, 4)$



very rough curves are all that you can do here

g) graph estimate $x(2) \approx 5.1$

from b): $x(2) = 1+4 - \frac{1}{2(1+4)^{3/2}} = 5 - \frac{1}{2(5)^{3/2}}$
 ≈ 4.956 close enough

arrows not easy to follow by hand

f) set $\frac{dx}{dt} = 0$ in DE: $0 + \frac{3}{1+2t} x = 5$ solve:

$x = \frac{5(1+2t)}{3} = \frac{5}{3} + \frac{10}{3}t$ straight line