MAT2705-01/06 07F Quiz 5 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

[You may use technology for row reductions, backsubstitutions and determinants.]

1. $v_1 = \langle 1, 2, 3, 2\rangle$, $v_2 = \langle 1, -1, 1, 0\rangle$, $v_3 = \langle 2, 1, 1, 2\rangle$, $v_4 = \langle 4, 2, 5, 4\rangle$, $v_5 = \langle 2, 4, 3, 4\rangle$

a) Express v_5 as a linear combination of the remaining 4 vectors, in the most general way. [Final answer: $v_5 = \ldots\, v_1 + \ldots$]

b) Check that this general linear combination that you find actually evaluates to v_5.

c) Find the independent linear relationships among these 4 vectors *v1*, *v2*, *v3*, *v4*. Write out these relationships individually.

2. Demonstrate that the following vectors are linearly independent:
$u_1 = \langle 1, 2, 3, 4\rangle$, $u_2 = \langle 2, 3, 4, 1\rangle$, $u_3 = \langle 3, 4, 1, 2\rangle$, $u_4 = \langle 4, 1, 2, 3\rangle$.

▶ solution

① a) $x_1\underline{v}_1 + x_2\underline{v}_2 + x_3\underline{v}_3 + x_4\underline{v}_4 = \underline{v}_5$

$$\begin{bmatrix} 1 & 1 & 2 & 4 \\ 2 & -1 & 1 & 2 \\ 3 & 1 & 1 & 5 \\ 2 & 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 4 & 2 \\ 2 & -1 & 1 & 2 & 4 \\ 3 & 1 & 1 & 5 & 3 \\ 2 & 0 & 2 & 4 & 4 \end{bmatrix} \xrightarrow[\text{maple}]{\text{rref}} \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 & x_4 \\ L & L & L & F \end{matrix} \end{array}$$

$x_1 + x_4 = 1 \to x_1 = 1-t$

$x_2 + x_4 = -1 \to x_2 = -1-t$

$x_3 + x_4 = 1 \to x_3 = 1-t$

$x_4 = t \to x_4 = t$

$$\boxed{\underline{v}_5 = (1-t)\underline{v}_1 - (1+t)\underline{v}_2 + (1-t)\underline{v}_3 + t\,\underline{v}_4}$$

b) $$(1-t)\begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} - (1+t)\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + (1-t)\begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix} + t\begin{bmatrix} 4 \\ 2 \\ 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 1-t-1-t+2-2t+4t \\ 2-2t+1+t+1-t+2t \\ 3-3t-1-t+1-t+5t \\ 2-2t+2-2t+4t \end{bmatrix} = \begin{bmatrix} (1-1+2)+(-1-1-2+4)t \\ (2+1+1)+(-2+1-1+2)t \\ (3-1+1)+(-3-1-1+5)t \\ (2+2)+(-2-2+4)t \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \\ 3 \\ 4 \end{bmatrix} \checkmark$$

c) $x_1\underline{v}_1 + x_2\underline{v}_2 + x_3\underline{v}_3 + x_4\underline{v}_4 = \underline{0}$ has solutions $(x_1, x_2, x_3, x_4) = (-t, -t, -t, t) = t(-1,-1,-1,1)$

so $\boxed{-\underline{v}_1 - \underline{v}_2 - \underline{v}_3 + \underline{v}_4 = 0}$

② $$\underline{A} = \langle \underline{u}_1 | \underline{u}_2 | \underline{u}_3 | \underline{u}_4 \rangle = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix}$$

$\xrightarrow{\text{maple}}$ 1) $\text{rref}(\underline{A}) = \underline{I}$, $\therefore A^{-1}$ exists $\therefore$ $\underline{A}\underline{x} = 0 \to \underline{x} = A^{-1}0 = \underline{0}$

so no linear relationships exist

$\xrightarrow{\text{maple}}$ 2) $|\underline{A}| = 160 \neq 0$, so $\underline{A}$ is invertible and the set of vectors is linearly independent