MAT2705-01/06 07F Quiz 1   Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. Write a differential equation that models the situation: "The time rate of change of the velocity v of a coasting motorboat is proportional to the square of the velocity." If you use common sense ("coasting motorboat"), can you say something about the sign of the proportionality constant (i.e., should the velocity increase or decrease?)?

2. $\dfrac{dy}{dx} = -2\,x\,e^{-x^2}, y(0) = 0.$

a) First find the general solution.
b) Then find the solution which satisfies the given initial condition.
c) Check that your result for part b) actually is a solution by backsubstituting.

▶ solution

1. $\dfrac{dv}{dt} \propto v^2 \rightarrow \boxed{\dfrac{dv}{dt} = k v^2}$

Most of us would assume $v \geq 0$ so v is actually the speed which should decrease so $k<0$:

$\boxed{\dfrac{dv}{dt} = -kv^2,\quad k>0}$

[however, if $v \leq 0$ we need the speed $-v$ to decrease so

$v<0$: $\dfrac{dv}{dt} = kv^2$, $k>0$] ← we need to always be careful about signs!

2. a) $\left[\dfrac{dy}{dx} = -2xe^{-x^2}\right] dx \rightarrow \quad dy = -2x\,e^{-x^2} dx \rightarrow$

$$\underbrace{\int dy}_{=\,y} = \int e^{\overbrace{-x^2}^{u}} \underbrace{(-2x\,dx)}_{du} = \int e^u du = e^u + C = e^{-x^2} + C$$

$\boxed{y = e^{-x^2} + C}$ general soln

b) $0 = y(0) = e^{-0^2} + C = 1 + C \rightarrow C = -1$

$\boxed{y = e^{-x^2} - 1}$ IVP soln

c) $\dfrac{dy}{dx} = e^{-x^2}(-2x) - 0 = -2xe^{-x^2}$ so backsubstituting into the DE:

$\dfrac{dy}{dx} = -2xe^{-2x}$

$-2xe^{-2x} = -2xe^{-2x}$ ✓