MAT2705-01/06 07F Final Exam Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). You are encouraged to use technology to check all of your hand results. ***You should use technology row reduction to solve 2x2 matrix equations to avoid fractional arithmetic errors which make the numbers much more complicated. Ditto for quadratic equations.***

1. $x'' + 13\,x' + 36\,x = 25\sin(3\,t),\ x(0) = 5,\ x'(0) = 5$.
a) Find the general solution by hand, showing all steps.
b) Find the initial value problem solution by hand, showing all steps.
c) In the actual solution to part b), what is the amplitude of the steady state part? Numerically find the value of *t* for which the (positive!) transient decays to 1 percent of this amplitude. Make a rough sketch of the full solution function and the steady state part together in an appropriate window showing this 1 percent time marked off on the axis.

2. A two mass, two spring system with parameters $m_1 = 1/2,\ m_2 = 1/12,\ k_1 = 3,\ k_2 = 1/2$ has the following equations of motion
$x_1'' = -7\,x_1 + x_2 + f_1\,,\ x_2'' = 6\,x_1 - 6\,x_2 + f_2,\ x_1(0) = 5,\ x_2(0) = 5,\ x_1'(0) = 0,\ x_2'(0) = 0$.
a) Rewrite this system of DEs and its initial conditions in matrix form for the vector variable $\vec{\boldsymbol{x}} = \langle x_1, x_2 \rangle$, identifying the coefficient matrix $\boldsymbol{A}$.
b) By hand showing all steps, find the standard eigenvectors $\boldsymbol{b}_1, \boldsymbol{b}_2$ produced by the solution algorithm with eigenvalues ordered in decreasing order. Evaluate the matrix $\boldsymbol{B} = \langle \boldsymbol{b}_1 | \boldsymbol{b}_2 \rangle$
and its inverse. On the grid provided, indicate by thick arrows both eigenvectors, extending them to labeled coordinate y_1, y_2 axes, and draw in the vector $\boldsymbol{x}(0)$ and its parallelogram projection onto those axes. Evaluate the new coordinates of $\boldsymbol{x}(0)$ using matrix methods. Do their values seem consistent with your drawing? Explain.
c) For certain applied forces proportional to the weight, one has $\langle f_1, f_2 \rangle = \langle 36, 36 \rangle$. Find the new equilibrium position by solving $\boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{f} = \boldsymbol{0}$ using matrix methods.
d) For the forcefree system $\boldsymbol{f} = \boldsymbol{0}$ find the general solution of the DE system by hand, showing all steps.
e) Find the solution which satisfies the initial conditions, by hand, showing all steps.
f) This has two modes: a "tandem" mode (same sign values of the two unknowns) and an "accordian" mode (opposite sign values of the two unknowns). What are the frequencies and periods of these two oscillating modes respectively (indicate which is which)? What is the common period of the periodic free motion of this system? Sketch the clearly labeled individual variables x_1, x_2 versus *t* for one period of this motion.
g) Now solve by hand the corresponding first order DE system replacing $\boldsymbol{x}''$ by $\boldsymbol{x}'$, using the same initial condition for $\boldsymbol{x}(0)$, showing all steps.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet stapled on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

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