MAT2705-02/03 06S Quiz 8   Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology to row reduce matrices and to find determinants and the (integer) roots of the characteristic equation.**

$$1.\ A=\begin{bmatrix} 9 & -4 & 0 \\ -6 & -1 & 0 \\ 6 & 4 & 3 \end{bmatrix}$$

a) Find the eigenvalues and eigenvectors of the matrix A following the full procedure: evaluate the characteristic equation $\det(A-\lambda I)=0$, solve it for the eigenvalues, then backsubstitute each into the linear system of equations whose solution leads to an eigenbasis for the eigenspace associated with that eigenvalue, scaling up your basis vectors so they have integer components. You need to report the characteristic equation, its solutions, and for each eigenvalue ~~the starting augmented matrix~~ and its rref form, from which by hand you must solve for the solution space basis. Label the basis vectors **b1, b2, b3**.

b) Finally augment them into a matrix B and use it to solve the linear system $B\,\mathbf{y}=\langle 1,-2,3\rangle$ in order to express the vector $\mathbf{x}=\langle 1,-2,3\rangle$ as an explicit linear combination of the eigenvectors: $\mathbf{x}=?\,\boldsymbol{b1}+?\boldsymbol{b2}+?\boldsymbol{b3}$.

▶ solution

my choice: ordered by increasing value.

a) $|A-\lambda I| = \begin{vmatrix} 9-\lambda & -4 & 0 \\ -6 & -1-\lambda & 0 \\ 6 & 4 & 3-\lambda \end{vmatrix} = -\lambda^3+11\lambda^2+9\lambda-99 = -(\lambda+3)(\lambda-3)(\lambda-11)=0 \to \lambda=-3,3,11$

$\lambda=-3$ $\quad A+3I = \begin{bmatrix} 12 & -4 & 0 \\ -6 & 2 & 0 \\ 6 & 4 & 6 \end{bmatrix} \longrightarrow \begin{array}{ccc} L & L & F \end{array} \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_1+\frac{1}{3}x_3=0 \to x_1=-\frac{1}{3}t$
$x_2+x_3=0$ $\quad x_2=-t$
$x_3=t$

$\vec{x}=\begin{bmatrix} -t/3 \\ -t \\ t \end{bmatrix} = t\begin{bmatrix} -1/3 \\ -1 \\ 1 \end{bmatrix}$ x3 $\quad \vec{b}_1=\begin{bmatrix} -1 \\ -3 \\ 3 \end{bmatrix}$

$\lambda=3$ $\quad A-3I = \begin{bmatrix} 6 & -4 & 0 \\ -6 & -4 & 0 \\ 6 & 4 & 0 \end{bmatrix} \longrightarrow \begin{array}{ccc} L & L & F \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_1=0$
$x_2=0$
$x_3=t$

$\vec{x}=\begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix}=t\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\vec{b}_2$

$\lambda=11$ $\quad A-11I = \begin{bmatrix} -2 & -4 & 0 \\ -6 & -12 & 0 \\ 6 & 4 & -8 \end{bmatrix} \to \begin{array}{ccc} L & L & F \end{array} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_1-2x_3=0$ $\quad x_1=2t$
$x_2+x_3=0$ $\quad x_2=-t$
$x_3=t$

$\vec{x}=\begin{bmatrix} 2t \\ -t \\ t \end{bmatrix}=t\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ $\vec{b}_3$

b) oops. ⓧ copied wrong initially

$B=\langle b_1|b_2|b_3\rangle = \begin{bmatrix} -1 & 0 & 2 \\ -3 & 0 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ (order free to choose)

another stupid mistake, dividing by 7 ⓧ but forgot to compensate initially

$By=x \to \quad y=B^{-1}x=\frac{1}{7}\begin{bmatrix} -1 & -2 & 0 \\ 0 & 7 & 7 \\ 3 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3/7 \\ 1 \\ 5/7 \end{bmatrix}$

I should redo this, but will I have time?

So $\boxed{\langle 1,-2,3\rangle = \frac{3}{7}\langle -1,-3,3\rangle + 1\langle 0,0,1\rangle + \frac{5}{7}\langle 2,-1,1\rangle}$

check: $= \frac{1}{7}\langle -3+0+10, -9+0-5, 9+7+5\rangle$
$= \langle \frac{7}{7}, -\frac{14}{7}, \frac{21}{7}\rangle = \langle 1,-2,3\rangle$

check
~~$=\langle -1+0+2, -1+0-1, 3+1+1\rangle$~~
~~$=\langle 1,-2,3\rangle$~~

not requested but a good idea