

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

$$2x_1 + 4x_2 + 6x_3 = 8$$

$$3x_1 + 5x_2 + 7x_3 = 9$$

a) Write down the coefficient matrix A, the RHS matrix b and the augmented matrix $C = \langle A | b \rangle$ for this linear system of equations.

b) Reduce this matrix to its ReducedRowEchelonForm by hand in 4 easy steps, annotating the MultiplyRow, AddRow, SwapRow operations you apply to each successive matrix in the process, using your own words or

the notation: $R_1 \rightarrow 3R_1, R_1 \rightarrow R_1 + 2R_1, R_1 \leftrightarrow R_2$. Hint: The first two columns should be: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

c) Write out the pair of equations that correspond to the reduced matrix. Identify the leading variables and the free variables and solve for the leading variables, assigning arbitrary parameters to the free variables. State your solution in the form:

$$x_1 = \dots, x_2 = \dots, x_3 = \dots$$

► solution

a) $A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$ $b = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \end{bmatrix} \xrightarrow{R_1 \rightarrow \frac{1}{2}R_1} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 5 & 7 & 9 \\ -3 & -6 & -9 & -12 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - 3R_1} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \rightarrow -R_2}$

initially miscopied! $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \boxed{\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{bmatrix}}$

c) $1x_1 + 0x_2 - 1x_3 = -2$
 $0x_1 + 1x_2 + 2x_3 = 3$

or $\boxed{\begin{matrix} x_1 - x_3 = -2 \\ x_2 + 2x_3 = 3 \end{matrix}} \rightarrow \begin{matrix} x_1 = -2 + x_3 = -2 + t \\ x_2 = 3 - 2x_3 = 3 - 2t \end{matrix}$

leading variables: x_1, x_2
 free variable: $x_3 \rightarrow x_3 = t$

so $\boxed{x_1 = -2 + t, x_2 = 3 - 2t, x_3 = t}$

Unrequested check: $2x_1 + 4x_2 + 6x_3 = 2(-2+t) + 4(3-2t) + 6(t) = -4 + 12 + (2-8+6)t = 8 \checkmark$

$3x_1 + 5x_2 + 7x_3 = 3(-2+t) + 5(3-2t) + 7(t) = -6 + 15 + (3-10+7)t = 9 \checkmark$