MAT2705-02/03 06S Test 3 TakeHome Answers

① $2q''+4q'+(\frac{1}{20})^{-1}q = E(t)$

$q''+2q'+10q=\frac{1}{2}E(t)$

a) $q''+2q'+10q=50$

LHS: $q\sim e^{rt} \to r^2+2r+10=0$

$r=\frac{-2\pm\sqrt{4-4\cdot10}}{2}=-1\pm3i$

$e^{rt}=e^{(-1\pm3i)t}=e^{-t}e^{\pm3it}$

$\xrightarrow{Re,Im}$ $e^{-t}\cos3t,\ e^{-t}\sin3t$

$\therefore\ q_h=e^{-t}(c_1\cos3t+c_2\sin3t)$

RHS: $D(50)=0 \to$ soln of $Dq=0 \leftrightarrow r=0$

gensoln: $q_p=c_3$

backsub: $\underbrace{c_3''}_{0}+2\underbrace{c_3'}_{0}+10c_3=50$

$c_3=\frac{50}{10}=5 \to q_p=5$

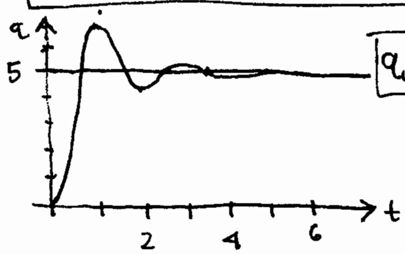
$$\boxed{q=q_h+q_p=\underbrace{e^{-t}(c_1\cos3t+c_2\sin3t)}_{\text{transient}}+\underbrace{5}_{\text{steady state}}}$$

$q'=-e^{-t}(c_1\cos3t+c_2\sin3t)+e^{-t}(-3c_1\sin3t+3c_2\cos3t)+0$

$q(0)=c_1+5=0 \to c_1=-5$

$q'(0)=-c_1+3c_2=0 \to c_2=\frac{1}{3}c_1=-\frac{5}{3}$

$$\boxed{q=-5e^{-t}(\tfrac{1}{3}\cos3t+\sin3t)+5}$$



$$\boxed{q_\infty=\lim_{t\to\infty}q=5}$$

b) $q''+2q'+10q=\underline{50e^{-t/10}}$

$r=-\frac{1}{10}$ $\quad(D+\frac{1}{10})(50e^{-t/10})=0$

gensoln: $q_p=c_3e^{-t/10}$

backsub:

$e^{-\frac{t}{10}}(\frac{c_3}{100}-\frac{c_3}{10}(2)+10c_3)=50e^{-t/10}$

$c_3(\frac{1-20+1000}{100})=50 \quad c_3=\frac{50\cdot100}{981}=\frac{5000}{981}\approx5.097$

$q_p=\frac{5000}{981}e^{-t/10}$

$q=q_h+q_p=e^{-t}(c_1\cos3t+c_2\sin3t)+\frac{5000}{981}e^{-t/10}$

$q'=-e^{-t}(c_1\cos3t+c_2\sin3t)+e^{-t}(-3c_1\sin3t+3c_2\cos3t)-\frac{500}{981}e^{-\frac{t}{10}}$

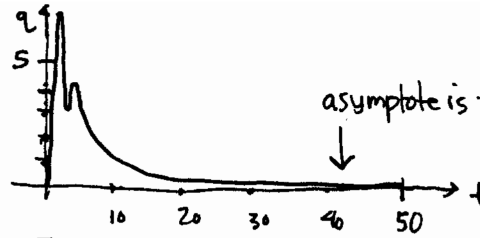
$q(0)=c_1+\frac{5000}{981}=0 \to c_1=-\frac{5000}{981}$

$q'(0)=-c_1+3c_2-\frac{500}{981}=0 \to c_2=\frac{1}{3}(c_1+\frac{500}{981})=\frac{-4500}{3(981)}=\frac{-1500}{981}$

$$\boxed{q=-\frac{500}{981}e^{-t}(10\cos3t+3\sin3t)+\frac{5000}{981}e^{-t/10}}$$

$$\boxed{q_\infty=\lim_{t\to\infty}q=0}$$

①b) continued



c) $q''+2q'+10q=\underline{50\cos3t}$

$r=\pm3i \to (D^2+9)(50\cos3t)=0$

gensoln: $10[q_p=c_3\cos3t+c_4\sin3t]$

$2[q_p'=-3c_3\sin3t+3c_4\cos3t]$

$1[q_p''=-9c_3\cos3t-9c_4\sin3t]$

$\text{LHS}_p=[(10-9)c_3+6c_4]\cos3t+[-6c_3+(10-9)c_4]\sin3t=50\cos3t$

$c_3+6c_4=50$

$-6c_3+c_4=0$

$\begin{bmatrix}1&6\\-6&1\end{bmatrix}\begin{bmatrix}c_3\\c_4\end{bmatrix}=\begin{bmatrix}50\\0\end{bmatrix}$

$\begin{bmatrix}c_3\\c_4\end{bmatrix}=\frac{1}{37}\begin{bmatrix}1&-6\\6&1\end{bmatrix}\begin{bmatrix}50\\0\end{bmatrix}=\begin{bmatrix}50/37\\300/37\end{bmatrix} \qquad q_p=\frac{50}{37}(\cos3t+6\sin3t)$

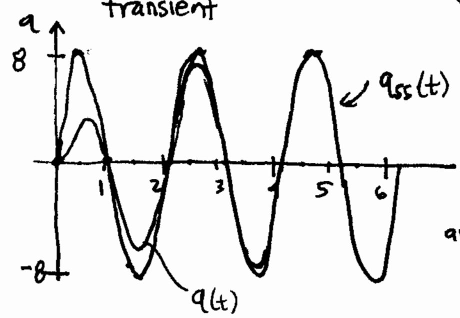
$q=q_h+q_p=e^{-t}(c_1\cos3t+c_2\sin3t)+\frac{50}{37}(\cos3t+6\sin3t)$

$q'=-e^{-t}(c_1\cos3t+c_2\sin3t)+e^{-t}(-3c_1\sin3t+3c_2\cos3t)+\frac{50}{37}(-3\sin3t+18\cos3t)$

$q(0)=c_1+50/37=0 \qquad c_1=-50/37$

$q'(0)=-c_1+3c_2+\frac{18\cdot50}{37}=0 \qquad c_2=\frac{1}{3}(c_1-\frac{18\cdot50}{37})=-\frac{19\cdot50}{3\cdot37}=\frac{-950}{111}$

$$\boxed{q=\underbrace{-\frac{50}{37}e^{-t}(\cos3t+\tfrac{19}{3}\sin3t)}_{\text{transient}}+\underbrace{\frac{50}{37}(\cos3t+6\sin3t)}_{\text{steady state}}}$$



indistinguishable after about 5 characteristic times = 5 units

amplitude $=\frac{50}{37}\sqrt{1+36}=\frac{50}{\sqrt{37}}\approx8.22$

d) $\underset{\text{standard form}}{1}q''+\underbrace{2}_{k_0}q'+\underbrace{10}_{\omega_0^2}q$ $\quad\left\{\begin{array}{l}\omega_0=\sqrt{10}\approx3.16\\ \tau_0=1/k_0=1/2=0.5\\ Q=\omega_0\tau_0=\sqrt{10}/2\approx1.58\end{array}\right.$

e) $q''+2q'+10q=\underline{50\cos\omega t}$

$r=\pm i\omega \to (D^2+\omega^2)(50\cos\omega t)=0$

↳ etc.. $10[q_p=c_3\cos\omega t+c_4\sin\omega t]$

$2[q_p'=-\omega c_3\sin\omega t+\omega c_4\cos\omega t]$

$1[q_p''=-\omega^2c_3\cos\omega t-\omega^2c_4\sin\omega t]$

$\text{LHS}_p=[(10-\omega^2)c_3+2\omega c_4]\cos\omega t$

$\underset{50}{}+\underset{0}{}[-2\omega c_3+(10-\omega^2)c_4]\sin\omega t$

$=50\cos\omega t$

$\begin{bmatrix}10-\omega^2&2\omega\\-2\omega&10-\omega^2\end{bmatrix}\begin{bmatrix}c_3\\c_4\end{bmatrix}=\begin{bmatrix}50\\0\end{bmatrix}$

$\begin{bmatrix}c_3\\c_4\end{bmatrix}=\frac{1}{(10-\omega^2)^2+4\omega^2}\begin{bmatrix}10-\omega^2&-2\omega\\2\omega&10-\omega^2\end{bmatrix}\begin{bmatrix}50\\0\end{bmatrix}=\frac{\begin{bmatrix}50(10-\omega^2)\\50(2\omega)\end{bmatrix}}{(10-\omega^2)^2+4\omega^2}$

steady state soln: $\boxed{q_p=\frac{50}{(10-\omega^2)^2+4\omega^2}[(10-\omega^2)\cos\omega t+2\omega\sin\omega t]}$

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(1) f) continued: $A(\omega) = \frac{50}{(10-\omega^2)^2+4\omega^2}\sqrt{(10-\omega^2)^2+4\omega^2}$

$= \frac{50}{\sqrt{(10-\omega^2)^2+4\omega^2}} = 50[(10-\omega^2)^2+4\omega^2]^{-1/2}$

$0 = A'(\omega) = 50(-\frac{1}{2})[(10-\omega^2)^2+4\omega^2]^{-3/2}[2(10-\omega^2)(-2\omega)+8\omega] =$

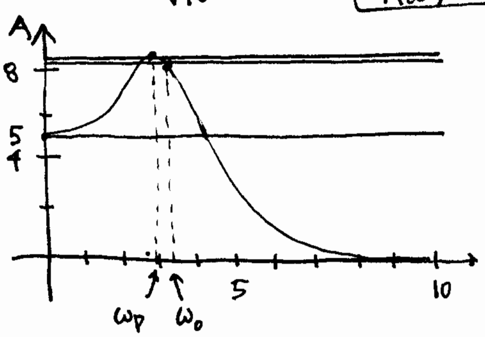
$= 100[\dots]^{-3/2}\,\omega(10-\omega^2-2) = 100[\dots]^{-3/2}\,\omega(8-\omega^2)$

$\to \omega = 0, \boxed{\sqrt{8} = 2\sqrt{2} = \omega_p \approx 2.83} \lessapprox \omega_0 = 3.16$

$A(2\sqrt{2}) = \frac{50}{\sqrt{(10-8)^2+4\cdot 8}} = \frac{50}{\sqrt{4\cdot 9}} = \frac{50}{6} = \boxed{\frac{25}{3} \approx 8.33}\ (A(\omega_p))$

$A(0) = \frac{50}{\sqrt{10^2}} = 5$ $\quad \boxed{\frac{A(\omega_p)}{A(0)} = \frac{5}{3} \approx 1.67} \gtrsim 1.58 = Q$

a little bigger but not by much



yes, all the results agree with the plot. the peak occurs just before the natural frequency with height just above 8. The zero frequency A axis intercept is 5.

(2) a) $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}' = \underbrace{\begin{bmatrix}-2&0&9\\2&-2&0\\0&2&-9\end{bmatrix}}_{A}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}, \quad \begin{bmatrix}x_1(0)\\x_2(0)\\x_3(0)\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

b) $\det(A-\lambda I) = \begin{vmatrix}-2-\lambda&0&9\\2&-2-\lambda&0\\0&2&-9-\lambda\end{vmatrix} \overset{\text{Maple}}{=} -\lambda^3-13\lambda^2-40\lambda = -\lambda(\lambda^2+13\lambda+40) = -\lambda(\lambda+5)(\lambda+8) = 0$

$\lambda = 0, -5, -8$

$\lambda = 0$: $A = \begin{bmatrix}-2&0&9\\2&-2&0\\0&2&-9\end{bmatrix} \to \begin{bmatrix}1&-1&0\\1&0&-9/2\\0&1&-9/2\end{bmatrix} \to \begin{bmatrix}1&-1&0\\0&1&-9/2\\0&1&-9/2\end{bmatrix}$

$\to \begin{array}{ccc}L&L&F\end{array}$ $\begin{bmatrix}1&0&-9/2\\0&1&-9/2\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ $\quad x_1 - 9/2\,x_2 = 0 \to x_1 = 9/2\,t$, $\quad x_2 - 9/2\,x_3 = 0 \quad x_2 = 9/2\,t$, $\quad x_3 = t$ $\quad \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = t\underbrace{\begin{bmatrix}9/2\\9/2\\1\end{bmatrix}}_{\vec{b}_1}$

$\lambda = -5$ $\quad A+5I = \begin{bmatrix}3&0&9\\2&3&0\\0&2&-4\end{bmatrix} \to \begin{bmatrix}1&0&3\\1&3/2&0\\0&1&-2\end{bmatrix} \to \begin{bmatrix}1&0&3\\0&3/2&-3\\0&1&-2\end{bmatrix} \to \begin{bmatrix}1&0&3\\0&1&-2\\0&0&0\end{bmatrix}$

$\begin{array}{ccc}L&L&F\end{array}$ $\begin{bmatrix}1&0&3\\0&1&-2\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ $\quad x_1 + 3x_3 = 0 \to x_1 = -3t$, $\quad x_2 - 2x_3 = 0 \to x_2 = 2t$, $\quad x_3 = t$ $\quad \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = t\underbrace{\begin{bmatrix}-3\\2\\1\end{bmatrix}}_{\vec{b}_2}$

$\lambda = -8$: $A+8I = \begin{bmatrix}6&0&9\\2&6&0\\0&2&-1\end{bmatrix} \to \begin{bmatrix}1&0&3/2\\1&3&0\\0&1&-1/2\end{bmatrix} \to \begin{bmatrix}1&0&3/2\\0&3&-3/2\\0&1&-1/2\end{bmatrix} \to$

$\begin{array}{ccc}L&L&F\end{array}$ $\begin{bmatrix}1&0&3/2\\0&1&-1/2\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ $\quad x_1 + 3/2\,x_3 = 0 \to x_1 = -\frac{3}{2}t$, $\quad x_2 - 1/2\,x_3 = 0 \to x_2 = \frac{1}{2}t$, $\quad x_3 = t$ $\quad \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = t\underbrace{\begin{bmatrix}-3/2\\1/2\\1\end{bmatrix}}_{\vec{b}_3}$

$B = \langle \vec{b}_1|\vec{b}_2|\vec{b}_3\rangle = \begin{bmatrix}9/2&-3&-3/2\\9/2&2&1/2\\1&1&1\end{bmatrix}$

$\vec{x} = B\vec{y} = y_1\vec{b}_1 + y_2\vec{b}_2 + y_3\vec{b}_3, \quad \vec{y} = B^{-1}\vec{x}$

$\vec{x}' = A\vec{x} \to B^{-1}(B\vec{y})' = B^{-1}A(B\vec{y})$

$\to \vec{y}' = \underbrace{(B^{-1}AB)}_{}\vec{y}$

$A_B = \begin{bmatrix}0&0&0\\0&-5&0\\0&0&-8\end{bmatrix}$ $\quad$ decoupled eqns:

$y_1' = 0 \qquad y_1 = c_1$

$y_2' = -5y_2 \qquad y_2 = c_2e^{-5t}$

$y_3' = -8y_3 \qquad y_3 = c_3e^{-8t}$

$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}9/2&-3&-3/2\\9/2&2&1/2\\1&1&1\end{bmatrix}\begin{bmatrix}c_1\\c_2e^{-5t}\\c_3e^{-8t}\end{bmatrix}$

$= \begin{bmatrix}\frac{9}{2}c_1 - 3c_2e^{-5t} - \frac{3}{2}c_3e^{-8t}\\ \frac{9}{2}c_1 + 2c_2e^{-5t} + \frac{1}{2}c_3e^{-8t}\\ c_1 + c_2e^{-5t} + c_3e^{-8t}\end{bmatrix}$ gen soln.

c) $\begin{bmatrix}x_1(0)\\x_2(0)\\x_3(0)\end{bmatrix} = B\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

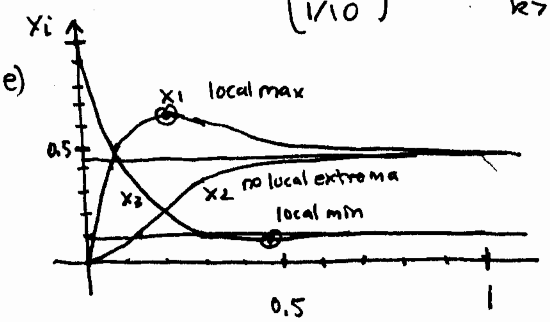
either RREF/Backsub $\langle B|\begin{bmatrix}0\\0\\1\end{bmatrix}\rangle$ or

$\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = B^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}1/10\\-3/5\\3/2\end{bmatrix}$

$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}\frac{9}{20} + \frac{9}{5}e^{-5t} - \frac{9}{4}e^{-8t}\\ \frac{9}{20} - \frac{6}{5}e^{-5t} + \frac{3}{4}e^{-8t}\\ \frac{1}{10} - \frac{3}{5}e^{-5t} + \frac{3}{2}e^{-8t}\end{bmatrix}$ ivp soln

d) $\vec{x}_\infty = \lim_{t\to\infty}\vec{x} = \begin{bmatrix}9/20\\9/20\\1/10\end{bmatrix}$ $\quad (e^{-kt} \to 0$, $k>0.)$

e)



f) $x_1 = \frac{9}{20} + \frac{9}{5}e^{-5t} - \frac{9}{4}e^{-8t}$

$x_1' = -9e^{-5t} + 18e^{-8t} = 0 \to e^{3t} = 2$

$t = \frac{1}{3}\ln 2 \approx 0.231$

$x_1(\frac{1}{3}\ln 2) = \frac{9}{20} + \frac{9}{5}e^{-\frac{5}{3}\ln 2} - \frac{9}{4}e^{-\frac{8}{3}\ln 2}$

$= \frac{9}{20} + \frac{9}{5}\cdot 2^{-5/3} - \frac{9}{4}\cdot 2^{-8/3} \approx 0.663$ $\quad$ looks right!

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(2f) continued.

$x_2 = \frac{9}{20} - \frac{6}{5}e^{-5t} + \frac{3}{4}e^{-8t}$

$x_2' = 6e^{-5t} - 6e^{-8t} = 0 \to e^{3t} = 1$, $t = 0$

no extremum for $t > 0$.

$x_3 = \frac{1}{10} - \frac{3}{5}e^{-5t} + \frac{3}{2}e^{-8t}$

$x_3' = 3e^{-5t} - 12e^{-8t} = 0 \to e^{3t} = 4$, $t = \frac{1}{3}\ln 4 \approx 0.462$

$x_3(\frac{1}{3}\ln 4) = \frac{1}{10} - \frac{3}{5\cdot 4^{5/3}} + \frac{3}{2\cdot 4^{8/3}} \approx 0.0777$ looks right!

g) x_1 comes down to 9/20 so

$1.01 \cdot \frac{9}{20} = x_1(t) \to$ solve numerically: $t \approx 1.192$ (maple)

x_2 comes up to 9/20 so

$0.99 \cdot \frac{9}{20} = x_2(t) \to$ solve numerically: $t \approx 1.113$ (maple)

x_3 comes up to $\frac{1}{10}$ so

$0.99 \cdot \frac{1}{10} = x_3(t) \to$ solve numerically: $t \approx 1.268$

(3) a) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \underbrace{\begin{bmatrix} 1 & -\frac{3}{2} \\ 3 & -2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b) $|A - \lambda I| = \begin{vmatrix} 1-\lambda & -3/2 \\ 3 & -2-\lambda \end{vmatrix} = (\lambda - 1)(\lambda + 2) + \frac{9}{2} = \lambda^2 + \lambda + \frac{5}{2} = 0$

$\lambda = \frac{-1 \pm \sqrt{1 - 4\cdot\frac{5}{2}}}{2} = \frac{-1 \pm \sqrt{-9}}{2} = -\frac{1}{2} \pm \frac{3}{2}i$

$\lambda = -\frac{1}{2} + \frac{3}{2}i$ $\quad A - \lambda I = \begin{bmatrix} 1 - (-\frac{1}{2} + \frac{3}{2}i) & -3/2 \\ 3 & -2 - (-\frac{1}{2} + \frac{3}{2}i) \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{2} - \frac{3}{2}i & -\frac{3}{2} \\ 3 & -\frac{3}{2} - \frac{3}{2}i \end{bmatrix} \to \begin{bmatrix} 1 & -\frac{1}{2} - \frac{1}{2}i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

L F

$x_1 - (\frac{1}{2} + \frac{i}{2})x_2 = 0$, $x_1 = (\frac{1}{2} + \frac{i}{2})t$ $\quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t \underbrace{\begin{bmatrix} \frac{1}{2} + \frac{i}{2} \\ 1 \end{bmatrix}}_{\vec{b}_1}$

$x_2 = t$

$\vec{b}_2 = \overline{\vec{b}_1}$, $\underline{B} = \begin{bmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ 1 & 1 \end{bmatrix}$

$B^{-1}AB = \begin{bmatrix} -\frac{1}{2} + \frac{3}{2}i & 0 \\ 0 & -\frac{1}{2} - \frac{3}{2}i \end{bmatrix} = A_B \to \boxed{\begin{matrix} \vec{x} = B\vec{y} \\ \vec{y} = B^{-1}\vec{x} \end{matrix}}$

$\vec{x}' = A\vec{x} \to B^{-1}(B\vec{y})' = B^{-1}A(B\vec{y})$

$\vec{y}' = A_B\vec{y}$

$y_1' = (-\frac{1}{2} + \frac{3}{2}i)y_1 \quad y_1 = c_1 e^{(-\frac{1}{2} + \frac{3}{2}i)t}$

$y_2' = (-\frac{1}{2} - \frac{3}{2}i)y_2 \quad y_2 = c_2 e^{(-\frac{1}{2} - \frac{3}{2}i)t}$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underline{B}\,\underline{y} = \begin{bmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 e^{(-\frac{1}{2} + \frac{3}{2}i)t} \\ c_2 e^{(-\frac{1}{2} - \frac{3}{2}i)t} \end{bmatrix}$

$= c_1 \underbrace{e^{(-\frac{1}{2} + \frac{3}{2}i)t} \begin{bmatrix} 1+i \\ 1 \end{bmatrix}} + c_2 e^{(-\frac{1}{2} - \frac{3}{2}i)t} \begin{bmatrix} 1-i \\ 1 \end{bmatrix}$

I should have redone this but it is important to realize everybody makes mistakes.

sloppy, forgot to divide by 2

$e^{-\frac{1}{2}t}(\cos\frac{3}{2}t + i\sin\frac{3}{2}t) \begin{bmatrix} \frac{1+i}{2} \\ 1 \end{bmatrix} = e^{-\frac{1}{2}t} \begin{bmatrix} \frac{\cos\frac{3t}{2} - \sin\frac{3}{2}t + i(\cos\frac{3}{2}t + \sin\frac{3}{2}t)}{2} \\ \cos\frac{3}{2}t + i\sin\frac{3}{2}t \end{bmatrix}$

$= e^{-\frac{1}{2}t} \begin{bmatrix} \frac{\cos\frac{3t}{2} - \sin\frac{3t}{2}}{2} \\ \cos\frac{3}{2}t \end{bmatrix} + i\, e^{-\frac{1}{2}t} \begin{bmatrix} \frac{\cos\frac{3t}{2} - \sin\frac{3}{2}t}{2} \\ \sin\frac{3t}{2} \end{bmatrix}$

so Re, Im parts are a real basis of soln space:

$$\boxed{\begin{aligned} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= c_1 e^{-\frac{t}{2}} \begin{bmatrix} \frac{\cos\frac{3t}{2} - \sin\frac{3t}{2}}{2} \\ \cos\frac{3t}{2} \end{bmatrix} + c_2 e^{-\frac{t}{2}} \begin{bmatrix} \frac{\cos\frac{3t}{2} + \sin\frac{3t}{2}}{2} \\ \sin\frac{3t}{2} \end{bmatrix} \\ &= \begin{bmatrix} e^{-\frac{t}{2}}\left(\frac{c_2 - c_1}{2}\sin\frac{3t}{2} + \frac{c_2 + c_1}{2}\cos\frac{3t}{2}\right) \\ e^{-\frac{t}{2}}\left(c_1\cos\frac{3t}{2} + c_2\sin\frac{3t}{2}\right) \end{bmatrix} \end{aligned}}$$

gen soln.

c) $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} \frac{c_1 + c_2}{2} \\ c_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \to \begin{matrix} c_1 = 1 \\ c_2 = 2 - c_1 = 1 \end{matrix}$

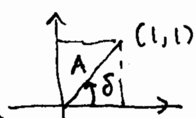
$$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e^{-\frac{t}{2}}\cos\frac{3t}{2} \\ e^{-\frac{t}{2}}(\cos\frac{3t}{2} + \sin\frac{3t}{2}) \end{bmatrix}}$$

IVP soln

d) $x_1 = e^{-\frac{t}{2}}(1\cos\frac{3t}{2} + 0\sin\frac{3t}{2})$

$= \underbrace{e^{-\frac{t}{2}}}_{A_1(t)}\cos\frac{3t}{2}$ $\quad A_1 = 1$, $\delta_1 = 0$ already in right form

$x_2 = e^{-\frac{t}{2}}(1\cos\frac{3t}{2} + 1\sin\frac{3t}{2})$

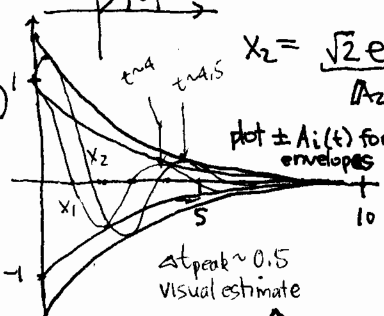


$A = \sqrt{1+1} = \sqrt{2}$

$\tan\delta = \frac{1}{1} = 1$, $\delta = \frac{\pi}{4}$ (45°)

$x_2 = \underbrace{\sqrt{2}e^{-t/2}}_{A_2(t)} \cos(\frac{3t}{2} - \frac{\pi}{4})$

e)



f) $\frac{3\Delta t}{2} = \frac{\pi}{4}$

$\Delta t = \frac{2}{3}(\frac{\pi}{4}) = \frac{\pi}{6} \approx$

≈ 0.524

(convert phase angle difference to time units)

$\frac{\pi}{4}/2\pi = \frac{1}{8}$ cycle

good agreement

x_2 lags behind x_1 since its peaks occur later in time, so x_1 is ahead (has its peaks earlier in time).