MAT2705-02/03 06F Test 1 Answers

① $x\frac{dy}{dx}+2-y=0$

a) $\frac{dy}{dx}=\frac{y-2}{x}$ separable standard form

$\int\frac{dy}{y-2}=\int\frac{dx}{x}$

$e^{\left[\ln|y-2| = \ln|x|+C_1\right]}$

$|y-2| = e^{\ln|x|+C_1} = e^{\ln|x|}e^{C_1} = xe^{C_1}$

$y-2 = \pm e^{C_1}x = C_2x$

$\boxed{y=2+C_2x}$

b) $\frac{dy}{dx}+\left(-\frac{1}{x}\right)y=-\frac{2}{x}$ standard linear form

$e^{\int-\frac{1}{x}dx}=e^{-\ln x}=x^{-1}$

$x^{-1}\left[\frac{dy}{dx}-\frac{1}{x}y=-\frac{2}{x}\right]$

$\frac{d}{dx}(yx^{-1}) = -2x^{-2}$

$yx^{-1}=\int-2x^{-2}dx=-2\left(\frac{x^{-1}}{-1}\right)+C_3$

$\boxed{y=2+C_3x}$

note: solutions agree with $C_2=C_3$.

c) $x\frac{d}{dx}(2+Cx)+2-(2+Cx)=0$

$x(0+C)+\underbrace{\cancel{2}-\cancel{2}}_{0}-Cx=0.$

$\underbrace{\cancel{Cx}-\cancel{Cx}}_{0}=0$

$0=0$ ✓

d) $y(1)=3$:

$3=y(1)=2+C(1)\rightarrow C=1$

$\boxed{y=2+x}$

e) $\longrightarrow$

② a) $V_0=160$ ft/s

$\frac{dv}{dt}=-32-\frac{v^2}{800}$ separable

$\underbrace{\int\frac{dv}{32+\frac{v^2}{800}}}_{5\arctan\left(\frac{v}{160}\right)}=\int-dt=-t+C_1$

solve for v:

$\arctan\left(\frac{v}{160}\right)=\frac{C_1}{5}-\frac{t}{5}$

$\frac{v}{160}=\tan\left(\frac{C_1}{5}-\frac{t}{5}\right)$

$\boxed{v=160\tan\left(\frac{C_1}{5}-\frac{t}{5}\right)}$ gen soln

$160=v(0)=160\tan\left(\frac{C_1}{5}\right)$

$1=\tan\left(\frac{C_1}{5}\right)\rightarrow\frac{C_1}{5}=\arctan 1=\frac{\pi}{4}$

$C_1=\frac{5}{4}\pi$

$\boxed{v=160\underbrace{\tan\left(\frac{\pi}{4}-\frac{t}{5}\right)}_{=0}}$ IVP soln

$=0\rightarrow v=0$

$\boxed{t=\frac{5\pi}{4}\approx 3.93\text{ s}}$

b) no drag:

$\frac{dv}{dt}=-32$

$\int dv=\int-32dt$

$v=-32t+C_1=160-32t=0$

$160=v(0)=C_1$

$\boxed{t=\frac{160}{32}=5\text{ s}}$

about 25% longer.