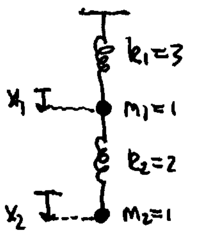
MAT2705-01/04 05S Quiz 9 Not! Print Name (Last, First) ____________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).




no gravity, pushpull on $m_2$ or hammer blow

deqs: scalar form
$$x_1'' = -5x_1 + 2x_2$$
$$x_2'' = 2x_1 - 2x_2 + A_0 \sin 2t$$

deqs: matrix form
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}'' = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ A_0 \sin 2t \end{bmatrix}$$
$$\underline{x}'' = \underline{A}\ \underline{x} + \underline{F}$$

new matrix form
$$\underline{y}'' = \underline{A}_B\, \underline{y} + \underline{B}^{-1}\underline{F}$$

$$\lambda_1 = -1, \lambda_2 = -6 \qquad \underline{B} = [\underline{E}_1\ \underline{E}_2] = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$
$$\underline{B}^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ -2/5 & 1/5 \end{bmatrix}, \quad \underline{A}_B = \underline{B}^{-1}\underline{A}\,\underline{B} = \begin{bmatrix} -1 & 0 \\ 0 & -6 \end{bmatrix}$$
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1/5 & 2/5 \\ -2/5 & 1/5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$\underline{x} = \underline{B}\ \underline{y} \qquad \underline{y} = \underline{B}^{-1}\underline{x}$$

Since the coefficient matrix $\underline{A}$ can be diagonalized, the new form of the deqs expressed in terms of $y_1$ and $y_2$ decouple from one another and can be solved independently.

a) Express the new deqs for $y_1$ and $y_2$ in scalar form and put them into the standard linear form (only terms involving unknowns on LHS with coefficient of 1 for highest derivative term).

b) Set $A_0 = 0$ and solve them, then impose the initial conditions $x_1(0) = 0 = x_2(0)$, $x_1'(0) = 0$, $x_2'(0) = -1$ corresponding to the masses initially at equilibrium and then mass 2 is hit upwards with a hammer imparting an initial velocity of 1 ft/sec.
To do this you need to solve the two systems: $\underline{x}(0) = \underline{B}\,\underline{y}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\underline{x}'(0) = \underline{B}\,\underline{y}'(0) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$.
Express your final results for $x_1, x_2$.

c) If you plot $x_1$ and $x_2$ can you estimate the largest displacements $(\max|x_1|, \max|x_2|)$ that either mass would make? Looking at the form of your expressions for $x_1$ and $x_2$ can you explain these limits?

d) Now ignore the homogeneous soln to explore what happens if the second mass is pushed and pulled by a sinusoidal driving function of frequency 2. With $A_0 > 0$ arbitrary (constant), use the method of undetermined coefficients to find particular solutions for $y_1$ and $y_2$ and then express your final results for $x_1, x_2$.