MAT2705-01/04 05S Quiz 8   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

Use technology for det, solve, rref; **A** = matrix([[5,3,-3], [0,3,0], [2,3,0]]).

a) Find the eigenvalues and eigenvectors of the matrix **A** following the full procedure: evaluate the characteristic equation det(**A**-λ**I**) = 0, solve it for the eigenvalues, then backsubstitute each into the linear system of equations whose solution leads to an eigenbasis for the eigenspace associated with that eigenvalue, scaling up your basis vectors so they have integer components. You need to report the characteristic equation, its solutions, and for each eigenvalue the starting augmented matrix and its rref form, from which by hand you must solve for the solution space basis. Label the basis vectors E1, E2, E3.
b) Finally augment them into a matrix B. Do your results agree with Maple? Explain.

a) $A = \begin{bmatrix} 5 & 3 & -3 \\ 0 & 3 & 0 \\ 2 & 3 & 0 \end{bmatrix}$ $\quad \det(A-\lambda I) = \det\left(\begin{bmatrix} 5-\lambda & 3 & -3 \\ 0 & 3-\lambda & 0 \\ 2 & 3 & -\lambda \end{bmatrix}\right) = -\lambda^3 + 8\lambda^2 - 21\lambda + 18$

$= -(\lambda-2)(\lambda-3)^2 = 0$

$\rightarrow \boxed{\lambda = 2, 3, 3}$

$\lambda = 2$: $\quad A - \lambda I = \begin{bmatrix} 3 & 3 & -3 \\ 0 & 1 & 0 \\ 2 & 3 & -2 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$: solve

LLF

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} x_1 = t \\ x_2 = 0 \\ x_3 = t \end{matrix}$

$\underline{x} = \begin{bmatrix} t \\ 0 \\ t \end{bmatrix} = t \boxed{\underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{E_1}}$

$\lambda = 3$: $\quad A - \lambda I = \begin{bmatrix} 2 & 3 & -3 \\ 0 & 0 & 0 \\ 2 & 3 & -3 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & \frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$: solve

LFF

$\begin{bmatrix} 1 & \frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad \begin{matrix} x_1 = -\frac{3}{2}t_1 + \frac{3}{2}t_2 \\ x_2 = t_1 \\ x_3 = t_2 \end{matrix}$

$\underline{x} = \begin{bmatrix} -\frac{3}{2}t_1 + \frac{3}{2}t_2 \\ t_1 \\ t_2 \end{bmatrix} = t_1 \begin{bmatrix} -\frac{3}{2} \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} \frac{3}{2} \\ 0 \\ 1 \end{bmatrix} \rightarrow$ double for convenience $\boxed{E_2 = \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} \quad E_3 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}}$

b) $\underline{B} = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

maple yields exactly the 3 vectors produced by the algorithm without doubling, in random order (which changes as you re-execute the command)

$\lambda = 2, 3, 3$