Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of the problem. Keep answers EXACT (not decimal approximations, if possible).

Use technology for det, solve, ref; $A = \text{matrix}([[5,3,-3], [0,3,0], [2,3,0]])$.

a) Find the eigenvalues and eigenvectors of the matrix $A$ following the full procedure: evaluate the characteristic equation $\det(A-\lambda I) = 0$, solve it for the eigenvalues, then back substitute each into the linear system of equations whose solution leads to an eigenvector for the eigenspace associated with that eigenvalue, scaling up your basis vectors so they have integer components. You need to report the characteristic equation, its solutions, and for each eigenvalue the starting augmented matrix and its ref form, from which by hand you must solve for the solution space basis. Label the basis vectors $E1, E2, E3$. 

b) Finally augment them into a matrix $B$. Do your results agree with Maple? Explain.

a) $A = \begin{bmatrix} 5 & 3 & -3 \\ 0 & 3 & 0 \\ 2 & 3 & -3 \end{bmatrix}$

$\det (A-\lambda I) = \det\begin{bmatrix} 5-\lambda & 3 & -3 \\ 0 & 3-\lambda & 0 \\ 2 & 3 & -3-\lambda \end{bmatrix} = -(\lambda+2)(\lambda-1)\lambda + 18 = 0$

$\Rightarrow (\lambda-2)(\lambda-1)(\lambda+3) = 0$

$\lambda = 2,1,-3$

$x = \begin{bmatrix} t \\ 0 \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} E1$

$x = \begin{bmatrix} \frac{1}{2} \frac{1}{2} \frac{1}{2} \\ \frac{1}{2} \frac{1}{2} \frac{1}{2} \end{bmatrix} t1 + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t2$

b) $B = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

$\lambda = 2, 3, -3$

Maple yields exactly the 3 vectors produced by the algorithm without doubling, in random order (which changes as you re-execute the command).