MAT2705-01/04 05S Quiz 5   Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

1. Calculate a determinant using technology (and state its value) to determine whether or not the three vectors are linearly independent:
a) **u** = [3,-1,2], **v** = [2,-3,5], **w** = ~~[4,5,-7]~~ [1,2,-3],
b) **u** = [1,-1,2], **v** = [3,0,1], **w** = [1,-2,2] .

2. Express **w** = [2,-7,9] as a linear combination of $\mathbf{u}_1$ = [1,-2,2], $\mathbf{u}_2$ = [3,0,1], $\mathbf{u}_3$ = [1,-1,2], if possible. Use technology to perform the necessary row reduction in one step and state the reduced matrix while solving this linear system of equations. Your final answer should be **w** = ... (an explicit linear combination of $\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3$).

3. $x_1 + 3x_2 + 3x_3 + 3x_4 = 0$, $2x_1 + 7x_2 + 5x_3 - x_4 = 0$, $2x_1 + 7x_2 + 4x_3 - 4x_4 = 0$.
a) Express this linear system of equations in matrix form: **A x** = **b**.
b) Use technology to reduce the augmented matrix in one step and state the reduced matrix before going on to finish solving the system by hand. Your final answer should be the expression for the column vector **x**.
c) Express the general solution in the form $x = t_1 \mathbf{v}_1 + ... + t_s \mathbf{v}_s$.
d) Let $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4\}$ be the columns of the coefficient matrix **A**. Based on your previous result, what independent relationships exist among these vectors, i.e., what independent linear combinations of these vectors equals the zero vector **0**: write them out in terms of the symbols $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4\}$.

1 a) $\begin{vmatrix} 3 & 2 & 1 \\ -1 & -3 & 2 \\ 2 & 5 & -3 \end{vmatrix} = 0$, therefore vectors not linearly ind.

b) $\begin{vmatrix} 1 & 3 & 1 \\ -1 & 0 & -2 \\ 2 & 1 & 2 \end{vmatrix} = -5 \neq 0$, therefore vectors are linearly ind.

2 $x_1 \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 9 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 3 & 1 \\ -2 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 9 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ -2 & 0 & -1 & -7 \\ 2 & 1 & 2 & 9 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{\text{solve}}$

$x_1 = 2$
$x_2 = -1$
$x_3 = 3$

so $\boxed{\vec{w} = 2\vec{u}_1 - \vec{u}_2 + 3\vec{u}_3}$ (check: $2\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2-3+3 \\ -4-3 \\ 4-1+6 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 9 \end{bmatrix}$ ✓)

3 a) $\underbrace{\begin{bmatrix} 1 & 3 & 3 & 3 \\ 2 & 7 & 5 & -1 \\ 2 & 7 & 4 & -4 \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\underline{b}}$

b) $\begin{bmatrix} 1 & 3 & 3 & 3 & 0 \\ 2 & 7 & 5 & -1 & 0 \\ 2 & 7 & 4 & -4 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{array}{c} \begin{array}{ccccc} L & L & L & F & \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 6 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 1 & 3 & 0 \end{bmatrix} \end{array} \rightarrow$

$x_4 = t$

$x_1 + 6x_4 = 0$ $\quad x_1 = -6t$
$x_2 - 4x_4 = 0$ $\quad x_2 = 4t$
$x_3 + 3x_4 = 0$ $\quad x_3 = -3t$

$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -6t \\ 4t \\ -3t \\ t \end{bmatrix} \overset{c)}{=} t \begin{bmatrix} -6 \\ 4 \\ -3 \\ 1 \end{bmatrix}$

d) $\boxed{-6\vec{u}_1 + 4\vec{u}_2 - 3\vec{u}_3 + \vec{u}_4 = \vec{0}}$ (check: $-6\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + 4\begin{bmatrix} 3 \\ 7 \\ 7 \end{bmatrix} - 3\begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix} = \begin{bmatrix} -6+12-9+3 \\ -12+28-15-1 \\ -12+28-12-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ✓)