MAT2705-01/04 05S Quiz 4 Print Name (Last, First) ____________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

1. The following fully reduced augmented matrix for a linear system of equations is $\mathbf{M} = \begin{bmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a) Write out the complete set of equations corresponding to this matrix in terms of the variables $x_1, x_2, \ldots$

b) Identify the leading variables (list them:{...}) and the free variables (list them: {...}).

c) Solve these equations and express the solution as a column matrix of unknowns $\mathbf{x}$, then rewrite it in the form $\mathbf{x} = t_1\, \mathbf{v}_1 + \ldots + \mathbf{c}$, where $\mathbf{v}_1, \mathbf{v}_2, \ldots$ are the constant column matrix coefficients of the free parameters, and $\mathbf{c}$ is the column matrix of numbers which do not have a free parameter in front of them.

d) Returning to the augmented matrix $\mathbf{M}$, write out the matrix form $\mathbf{A}\,\mathbf{x} = \mathbf{b}$ of the corresponding linear system of equations.

e) Delete the last column of the matrix $\mathbf{M}$ to form the matrix $\mathbf{N}$. What is the linear system of equations that corresponds to $\mathbf{N}$ as an augmented matrix and what is its solution? Explain.

2. Check that the following two matrices satisfy the condition that they be inverses: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & -5 & -3 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 1 \\ 4 & -5 & -2 \end{bmatrix}$.

① a)
$$\begin{aligned} x_2 + 2x_3 &= 3 \\ x_4 &= 4 \\ 0 &= 0 \end{aligned}$$

b) Leading variables: $\{x_2, x_4\}$

free variables: $\{x_1, x_3\}$

c) $x_1 = t_1$, $x_3 = t_2$ backsub & solve:

$x_2 = 3 - 2x_3 = 3 - 2t_2$

$x_4 = 4$

so $(x_1, x_2, x_3, x_4) = (t_1, 3-2t_2, t_2, 4)$

or

$$\underline{x} = \begin{bmatrix} t_1 \\ 3-2t_2 \\ t_2 \\ 4 \end{bmatrix} = t_1 \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\underline{v_1}} + t_2 \underbrace{\begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}}_{\underline{v_2}} + \underbrace{\begin{bmatrix} 0 \\ 3 \\ 0 \\ 4 \end{bmatrix}}_{\underline{c}}$$

d)
$$\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$$

e) $\underline{N}$:
$$\begin{aligned} x_2 + 2x_3 &= 0 \\ 0 &= 1 \\ 0 &= 0 \end{aligned}$$
← inconsistent system, no solution

②
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & -5 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 1 \\ 4 & -5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot1 + 0(-2) + 0\cdot4 & 1\cdot0 + 0\cdot3 + 0(-5) & 1\cdot0 + 0\cdot1 + 0(-2) \\ 0\cdot1 + 2(-2) + 1\cdot4 & 0\cdot1 + 2(3) + 1(-5) & 0\cdot0 + 2\cdot1 + 1(-2) \\ 2\cdot1 - 5(-2) - 3\cdot4 & 2\cdot0 - 5(3) - 3(-5) & 2\cdot0 - 5\cdot1 - 3(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -4+4 & 6-5 & 2-2 \\ 2+10-12 & -15+15 & -5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

product is the identity matrix

NOTE:

Many did not read the words carefully — check the condition NOT derive its solution. Once you have a solution it is always useful to check it by backsubstitution into the equation it solves.

condition: $AA^{-1} = I$.