

- 1. turn on forcing function

$$\text{deq} := \left(\frac{d^2}{dt^2} y(t) \right) + 60 \left(\frac{d}{dt} y(t) \right) + 800 y(t) = 48 - 48 e^{(-10t)}$$

$$\text{inits} := y(0) = 0, D(y)(0) = -1$$

- Find the general solution using the method of undetermined coefficients.
- Find the solution of the initial value problem.
- Plot this function in an appropriate window showing all of its behavior clearly, starting at $t = 0$.
- Find the numerical value of t at which $y(t)$ has its absolute minimum on the interval $t = 0 .. \infty$, by numerically solving the condition that its derivative is zero with technology. Does this agree with your plot?
- What is the asymptotic value of $y(t)$ for large t ? Does this agree with your plot?

- 2. earthquake

A building consists of two floors. The first floor is attached rigidly to the ground, and the second floor is of mass $m = 1000$ slugs (ft-lb-s units) and weighs 16 tons (32,000 lbs). The elastic frame of the building behaves as a spring that resists horizontal displacements of the second floor with a Hooke's law constant $K = 10,000$ lbs/ft and with a damping constant $c = 2000$ lbs/(ft/s). Assume that in an earthquake the ground oscillates horizontally with amplitude A_0 and frequency ω , resulting in an external horizontal force $F(t) = m A_0 \omega^2 \sin(\omega t)$ on the second floor. The relative horizontal displacement of the second floor with respect to the first satisfies:

$$\text{deq} := m \left(\frac{d^2}{dt^2} x(t) \right) + c \left(\frac{d}{dt} x(t) \right) + K x(t) = m A_0 \omega^2 \sin(\omega t)$$

$$[m, c, K] = [1000, 2000, 10000]$$

$$\text{inits} := x(0) = 0, D(x)(0) = 0$$

- Express the deq in standard form with a coefficient of 1 for the highest derivative term.
- Identify the values of the natural frequency ω_0 and natural decay constant k_0 for the system exactly and numerically.
- What are the corresponding values of the period $T = 2\pi/\omega_0$ and decay time τ_0 exactly and numerically, and what is the value of the quality factor $Q = \omega_0 \tau_0$ exactly and numerically? What does this say about the underdamped / critically damped / overdamped nature of the system?
- Derive the homogeneous solution (transient), showing all steps.
- For $\omega = 3$, $A_0 = 1/4$ ft = 3 in, find the particular solution (steady state solution) using the method of undetermined coefficients, showing all steps. What is its amplitude in inches?
- Now solve the initial value problem for this case.
- Find the particular function for arbitrary $\omega > 0$, showing all steps, and evaluate its amplitude $A(\omega)$.
- Plot $A(\omega)$ for $\omega > 0$ in an appropriate viewing window.
- Use calculus to find the exact and numerical value of ω at which this amplitude has a peak

(warning, for this problem the starting formula for $A(\omega)$ has an extra factor of ω^2 compared to the previous resonance discussions). What is the numerical value of the amplitude at this value of ω in inches? Do these values agree with your plot? Explain.

3. linsys 1

$$\text{deqs} := \frac{d}{dt} x_1(t) = -x_1(t), \frac{d}{dt} x_2(t) = x_1(t) - 2x_2(t), \frac{d}{dt} x_3(t) = 2x_2(t) - 3x_3(t)$$

$$\text{inits} := x_1(0) = 27, x_2(0) = 0, x_3(0) = 0$$

- Write this system in matrix form, i.e., for the vector variable $\mathbf{x} = \text{transpose}([x_1, x_2, x_3])$.
- Use the eigenvector approach to find its general solution, showing all steps.
- Find the IVP solution, showing all steps (except the row operations of the matrix reduction).
- What is the largest value $x_3(t)$ ever assumes for $t \geq 0$? (Give its numerical value.)

4. linsys2

$$\text{deqs} := \frac{d}{dt} x_1(t) = x_1(t) - 5x_2(t), \frac{d}{dt} x_2(t) = x_1(t) - x_2(t)$$

$$\text{inits} := x_1(0) = 1, x_2(0) = -1$$

- Use the eigenvector method to solve this IVP, showing all steps (except matrix row ops).
- What is the frequency and period of these solutions?
- Express the two variables in phase-shifted cosine form in terms of their amplitudes and phase shifts.
- What is the ratio of their amplitudes, what is the difference of their phase shifts in radians and then in degrees? Which one is ahead of the other in time?
- Make a plot of these functions showing 2 complete oscillations starting at $t = 0$. Does your response to part d) agree with your plot? Explain.

Instructions. Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers exact (no decimal approximations, if possible).

READ LONG INSTRUCTIONS at website or in paper copy.

After you are done, sign and date:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date: