Technology should be used to row reduce matrices in a single step (state initial and final matrices in your work), take determinants if necessary, or find exact roots of polynomials.

⊟ 1. linear system

a) Express the vector $[0, 3, 5]$ as a linear combination of the vectors {
$[1, -1, 1]$, $[1, 0, -4]$, $[0, 2, 0]$ }.
State the matrix equation $\mathbf{A}\,\mathbf{x} = \mathbf{b}$, its augmented matrix and its fully reduced form, and its solution, of the linear system of equations required to perform this task. Be sure your final answer is in the form: $[0,3,5] =$ <an explicit linear combination of the given vectors>.
b) Can any vector in R^3 be expressed as a linear combination of these vectors or not? Explain why or why not in words and back up any claims you make.

⊟ 2. IVP

$$deq := (\mathrm{D}^{(3)})(y)(x) + (\mathrm{D}^{(2)})(y)(x) + 4\,\mathrm{D}(y)(x) + 4\,\mathrm{y}(x) = 0$$

$$inits := \mathrm{y}(0) = 0,\ \mathrm{D}(y)(0) = 3,\ (\mathrm{D}^{(2)})(y)(0) = 5$$

a) Find the general solution of this deq, explaining all details of its derivation.
b) Find the solution of the initial value problem posed by the deq with the given initial conditions. Show all work. You may refer to the results of problem 1.
c) By backsubstitution into the deq, show that your solution to part b) satisfies that deq.

⊟ 3. another linear system

The balance equations for the reaction $n1\ CH_4 + n2\ O_2 \rightarrow n3\ CO_2 + n4\ C_2 H_2 + n5\ H_2 O$ are:

$$n1 - n3 - 2\,n4 = 0$$
$$4\,n1 - 2\,n4 - 2\,n5 = 0$$
$$2\,n2 - 2\,n3 - n5 = 0$$

a) Check that $[n1, n2, n3, n4, n5] = [4, 3, 0, 2, 6]$ satisfies these equations (revealing another more complicated oxidation reaction that is possible with the same reactants as in our chemical reaction example worksheet but having one extra product from the reaction).
b) Find the general solution of this system of equations (show your work as in problem 1) and find the basis vectors {$\mathbf{v1}$,$\mathbf{v2}$} which are coefficients of the arbitrary parameters in your general solution: $\mathbf{n} = t1*\mathbf{v1}+t2*\mathbf{v2}$, showing all the steps along the way. Then double these vectors to get integer component basis vectors; call them $\mathbf{u1}$ and $\mathbf{u2}$.
By chance one of these vectors corresponds directly to the simple oxidation reaction in our example chemical reaction worksheet (all its entries are nonnegative, as must be the case for integer solutions of the balance equations to describe a chemical reaction with these reactants and products).
c) [*If you are squeezed for time, consider this part optional.*] Express the vector $\mathbf{u3} =$
$[4, 3, 0, 2, 6]$ as a linear combination of these two vectors: $\mathbf{u3} = ?\mathbf{u1} + ?\mathbf{u2}$. Show your derivation as in problem 1.

On reverse side, see long instructions, date and sign pledge when finished.

Math Exam Rules

READ THESE INSTRUCTIONS CAREFULLY

This test is not about just getting "the right answer", but also presenting and communicating well the process which leads to the results requested in each part of every problem, as well as your understanding of the course content and its vocabulary. [This is good practice for learning how to communicate technical results to other people in a workplace environment.] Unless specifically requested, no results here may be *justified* using technology — a reasoned explanation supported by mathematical facts is always required and cannot be substituted by a technology result. [This course is trying to develop some elementary mathematical thinking skills which help you understand why you are seeing what you see in a technology screen.] However, you are encouraged to use MAPLE (or calculators) to check every result you derive by hand. Come talk to me if you get stuck on any problem or are confused. [For a take home exam, no collaboration is allowed but you may consult your textbook, your notes and my handouts.]

Show **<u>all</u>** work and answers, ***including indications of mental steps***, on the lined paper provided. If you copy over work, be sure you include everything. Put your name on each sheet and clearly label continuations of problems from one sheet to another. **LABEL** and **SEPARATE** clearly each part of each problem and **BOX** each short final response requested (and nothing else). Cross out abandoned work not to be considered.

Use proper mathematical notation: "symbol" = "expression representing symbol" = "...", introducing symbols if necessary. Don't misuse equal signs, and don't write down unidentified expressions, but do link expressions which are equal with equal signs, while using arrows or colons to link expressions which are not equal but are related by some step. Give **EXACT ANSWERS**, not decimal approximations, *unless the context warrants it*, but first give the exact result in any case. Always simplify results. **Math is case sensitive: always be consistent with your upper and lower case letters used as symbols.**

Take home test: If you use (computer) technology to check your work, print out your worksheet with comments, handwritten if necessary, labeled by problem, and attach it to your test after your hand work.

When you have completed the exam, please read and sign the dr bob integrity pledge and attach it to your answer sheets (staple take home test) as a cover page, first side face up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date: