

MAT 2705-01/04 OSS Test 1 Answers

① $\frac{dy}{dx} = 6e^{2x}e^{-y}$ separable

$e^y dy = 6e^{2x} dx$

$\int e^y dy = 6 \int e^{2x} dx$

$e^y = 6\left(\frac{e^{2x}}{2}\right) + C$

$y = \ln e^y = \ln(3e^{2x} + C)$ gen soln.

$0 = y(0) = \ln(3e^0 + C)$

$1 = e^0 = e^{\ln(3+C)} = 3+C$

$\rightarrow C = -2 \rightarrow$

$y = \ln(3e^{2x} - 2)$ IVP soln.

② a) $\frac{dy}{dx} = 2y + e^{-x}$ standard linear form

$e^{-2x} \left(\frac{dy}{dx} - 2y \right) = e^{-3x}$
 $\int -2dx = -2x$
 e^{-2x}

$\frac{d}{dx}(ye^{-2x}) = e^{-2x}e^{-x} = e^{-3x}$

$ye^{-2x} = \int e^{-3x} dx = \frac{e^{-3x}}{-3} + C$

$y = e^{2x} \left(-\frac{1}{3}e^{-3x} + C \right) = -\frac{1}{3}e^{-x} + Ce^{2x}$ gen soln.

b) $1 = y(0) = -\frac{1}{3}e^0 + Ce^0$

$1 = -\frac{1}{3} + C \rightarrow C = \frac{4}{3}$

$y = -\frac{1}{3}e^{-x} + \frac{4}{3}e^{2x}$ IVP soln.

$\frac{d}{dx} \left(-\frac{1}{3}e^{-x} + \frac{4}{3}e^{2x} \right) = 2 \left(-\frac{1}{3}e^{-x} + \frac{4}{3}e^{2x} \right) + e^{-x}$
 $+ \frac{1}{3}e^{-x} + \frac{8}{3}e^{2x} = -\frac{2}{3}e^{-x} + e^{-x} + \frac{8}{3}e^{2x}$
 $= \frac{1}{3}e^{-x} + \frac{8}{3}e^{2x} \checkmark$

see diagram below

③ a) $\frac{dT}{dt} = -k(T-72)$ separable or linear

$\int \frac{dT}{T-72} = -\int k dt$ separable & integrate

$\ln|T-72| = -kt + C_1$
 $|T-72| = e^{-kt+C_1} = e^{C_1} e^{-kt}$
 $T-72 = (\pm e^{C_1}) e^{-kt}$

$T = 72 + C e^{-kt}$ gen soln

b) $32 = T(0) = 72 + C e^0 = 72 + C \rightarrow C = -40$

$T(t) = 72 - 40 e^{-kt}$ IVP soln.

c) $52 = T(\frac{1}{2}) = 72 - 40 e^{-k/2}$

$-20 = -40 e^{-k/2}$

$\frac{1}{2} = e^{-k/2}$

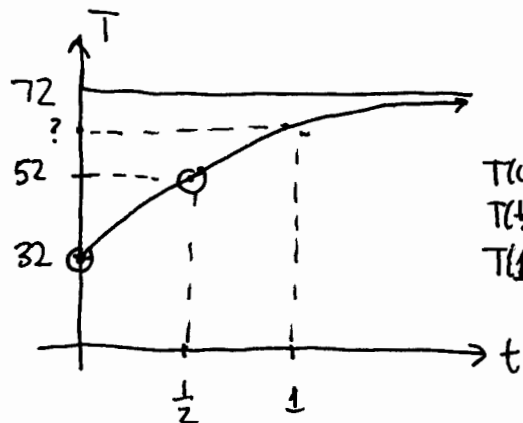
$2 = e^{k/2}$

$\ln 2 = k/2 \rightarrow k = 2 \ln 2 = \ln 4 \approx$

d) $T(1) = 72 - 40 e^{-k} = 72 - 40 e^{-\ln 4} = 72 - 40(4)^{-1} = 72 - 10 = 62 \rightarrow 62^\circ F$

The chicken is $62^\circ F$ one hour after being placed in the camp tent.

temperature (of course!)



$T(0) = 32$
 $T(\frac{1}{2}) = 52$
 $T(1) = ?$