MAT1505-05 04S Quiz 7 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$$

a) Use the ratio test to find the radius R of convergence of this power series (such that the series converges for $|x| < R$ and diverges for $|x| > R$).

b) Check the endpoints of the open interval $-R < x < R$ for convergence in order to state the complete interval of convergence in this same notation. Explain your claims.

c) $f(\frac{1}{2})$ is an alternating series so you can easily estimate the truncation error. What is the least number n for which the partial sum S_n up to the nth power term gives the value of $f(\frac{1}{2})$ to 3 decimal place accuracy? Give S_n to at least 6 decimal places.

d) The exact value is $f(\frac{1}{2}) = \arctan(\frac{1}{2})$. What is the value of the remainder $f(\frac{1}{2}) - S_n$ for the n you picked in part c)? Is it smaller in absolute value than your estimate in part c)?

a) $a_n = (-1)^n \dfrac{x^{2n+1}}{2n+1}$ $\qquad \left|\dfrac{a_{n+1}}{a_n}\right| = \dfrac{\dfrac{|x|^{2(n+1)+1}}{2(n+1)+1}}{\dfrac{|x|^{2n+1}}{2n+1}} = \dfrac{|x|^{2n+3}}{|x|^{2n+1}}\left(\dfrac{2n+1}{2n+3}\right) = |x|^2\left(\dfrac{2n+1}{2n+3}\right)$

$\lim_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} |x^2|\left(\dfrac{2n+1}{2n+3}\right) = |x^2| < 1$ for convergence $\rightarrow$ $|x| < \boxed{1 = R}$

b) $x = 1$: $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1}$ convergences by the alternating series test

$x = -1$: $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(-1)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{(-1)}{2n+1}$ also converges by alt series test.

so interval of convergence: $\boxed{-1 \le x \le 1}$

c) $f(\frac{1}{2}) = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(\frac{1}{2})^{2n+1}}{2n+1} = \frac{1}{2} - \frac{1}{3}\frac{1}{2^3} + \frac{1}{5}\frac{1}{2^5} - \frac{1}{7}\frac{1}{2^7} \Big| + \frac{1}{9}\frac{1}{2^9} - \cdots$

$\approx \underbrace{.00050 - .04167 + .00625 - .00111}_{.463467} + \underbrace{.000217}_{< .5\times 10^{-3}}$

cutoff after $n=3$

$\arctan(\frac{1}{2}) \approx .463647$

$\arctan(\frac{1}{2}) - S_3 \approx \boxed{.00018} < .00021$ estimate ✓