MAT1505-05 04S Test 3 Takehome Print Name (Last, First) ____________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers exact (no decimal approximations, if possible). [See long instructions on reverse].

1 a) By manipulating the geometric series formula, find a power series representation of $f(x) = \frac{x}{8-x^3}$ and determine its interval of convergence.

b) Is your interval of convergence consistent with a plot of f (if f has a vertical asymptote, its series much diverge there)? Explain.

2 $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2 5^n}$ Find the radius R of convergence and the complete interval of convergence for this series, justifying all your claims.

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3 $\lim_{x\to 0} \frac{1-\cos x}{1+x-e^x}$ a) Use series to evaluate this limit (you may assume the formulas for the Taylor series of $\cos x$ and e^x).

b) Use L'Hopital's rule to evaluate this limit, being careful to check its validity with each application.

c) If you plot this expression as a function of x and zoom in on $x=0$, is your graph consistent with your exact result? Explain.

4 $\int_0^{0.5} x^2 e^{-x^2}\,dx$ a) Use series to approximate this to 3 decimal places ($|\text{error}| < \frac{1}{2}\times 10^{-3}$), explicitly using the alternating series error estimate. What is the minimum number of terms n needed to achieve this accuracy and what is their sum to 6 decimal places?

b) Evaluate this integral numerically with technology — report your result to 6 decimal places. Use this to compute the error in your 6 digit value for S_n in part a). Is this error consistent with your stated error bound of $\frac{1}{2}\times 10^{-3}$?

c) Use the Simpson error formula to estimate the maximum error in using the $n=2$ Simpson rule to approximate this integral and give the value of this approximation. Show all details. [Use technology to plot $|f^{(4)}(x)|$ over $[0,0.5]$ to estimate its largest value.]

5 a) $f(x)=(1+x)^{-2}$. Use the Taylor series formula to evaluate the first 4 terms of the Taylor series for $f(x)$ at $x=0$.

b) Generalize the pattern you see established for these 4 terms to a sigma notation formula for the complete series, ie., get a formula for the nth term.

c) Show that you get the same result by term by term differentiation of the identity $(1+x)^{-1} = \sum_{n=0}^{\infty}(-x)^n$, since $(1+x)^{-2} = -\frac{d}{dx}(1+x)^{-1}$ (reverse the sign of course!).

d) The gravitational force F on a mass m at a height h above the surface of the Earth (radius R) is $F = \frac{mgR^2}{(R+h)^2}$. Show that this can be rewritten as $F = mg(1+\frac{h}{R})^{-2} = mg\, f(\frac{h}{R})$. Then use the first 2 terms of your Taylor series for f with $x=h/R$ to approximate F for values of h small compared to R. What are the formulas for each term separately?

e) The first term is the familiar force of gravity at the Earth's surface. Since the series is alternating, the second term can be used to estimate the maximum error in using only the first term as "the gravitational force." At what value of h does this error exceed 1% (of the first term) by this estimate if $R = 6371$ km?