If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after $t$ hours is $n = f(t) = 100 \cdot 2^{t/3}$

a) Find the inverse of this function and explain its meaning in a complete English sentence.

b) When will the population reach 50,000? Give the exact formula and the decimal approximation to 3 significant figures, with units.

c) Make a completely labeled graph that illustrates all of this information.

\[
\begin{align*}
\text{a) } n &= 100 \cdot 2^{t/3} = f(t) \\
\ln \left( \frac{n}{100} \right) &= \frac{t}{3} \\
\ln \left( \frac{50000}{100} \right) &= \frac{3}{\ln 2} \\
t &= \frac{3}{\ln 2} \ln \left( \frac{50000}{100} \right) \\
&\approx 26.897 \text{ hours}
\end{align*}
\]

This gives the number of hours it takes for the population to reach the value $n$.

\[\text{NOTE: once you have the inverse function, there is no need to resolve the original equation for a particular}\]
\[\text{n value like 50,000!}\]

\[\text{one plot cannot show the small scale starting behavior and the large scale growth}\]
\[\text{UNLESS we distort the graph to make a sort of "caricature":}\]

\[\text{NOTE: In solving for the inverse function, it makes no sense to interchange the variable names in the equation because each carries a}\]
\[\text{distinct meaning with different units.}\]