

**1. linear approximation**

- a) Find the linear approximation  $L(x)$  to the function  $f(x) = \ln(1 + x)$  at the reference point  $x = 0$ .
- b) How is the graph of  $f$  related to the graph of the  $\ln$  function? Make a rough hand graph of  $f$  and  $L$  on the interval  $x = -1..1$ . Is the linear approximation too high or too low? Explain why based on second derivative information.
- c) Use the linear approximation to approximate  $\ln(1.1)$  and evaluate the actual error = approx - exact. [ $1.1 = 1+0.1!$ ]
- d) Plot the function error  $L(x) - f(x)$  with technology by trial and error on the horizontal window and find the interval where this error is less than 0.1 in absolute value, reporting its endpoints to 2 decimal place accuracy. Make a completely labeled sketch of what you see. Is this interval compatible with the error you found in part c)?

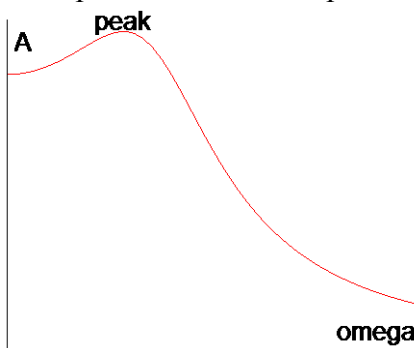
**2. related rates, differentials**

- a) The mechanics at CarTalk Central are boring a 6-in-deep cylinder to fit a new piston. The machine they are using increases the cylinder's radius one-thousandth of an inch every 3 minutes. How rapidly is the cylinder volume (recall  $V = \pi r^2 h$ ) increasing when the bore (diameter) is 3.800 in? Give your result to 3 significant figures (number of digits, not number of digits after the decimal point).
- b) Starting with a bore diameter of 3.800 in, if the radius increases by .005 inches, use the differential approximation to evaluate the approximate increase in the volume. What are the approximate percentage increases in radius and volume?

**3. max/min function**

$$A := \omega \rightarrow \frac{1}{\sqrt{\omega^4 - 16\omega^2 + 256}}$$

- a) Find the exact and 3 significant figure (number of digits, not number to right of decimal point) accurate values of the frequency  $\omega$  ( $0 \leq \omega$ ) and amplitude  $A$  of this amplitude-versus-frequency resonance response function for a damped oscillator at its peak:



- b) Explain why this has to be the global maximum, using derivative information.

#### 4. max/min word problem

Consider two real numbers between 0 and 1 (inclusive) whose sum is 1. How should they be chosen (exactly, not decimal numbers) so that the cube of the first plus 4 times the cube of the second:

- a) is maximum (and what is that maximum value?)
- b) is minimum (and what is that minimum value?)

Make sure your English sentence responses to each question are unambiguous to the reader without referring to your derivation. Support your conclusions by hand work as though you did not have technology to make a graph. Then confirm your work with a rough hand sketch from a technology graph, labeling all key points by both coordinates.

#### 5. roadmap graphing

Consider the graph  $y = f(x)$  for the function [note:  $\exp(-x/2)$  not  $\exp(-1/(2x))$  ]

$$f := x \rightarrow (1 - x^2) e^{(-1/2x)}$$

- a) Evaluate the  $x$  and  $y$  intercepts and **state** and **evaluate** the limits of  $f$  as  $x$  approaches  $+\infty/-\infty$  (using L'Hopital's rule when appropriate). Does  $f$  have any horizontal asymptotes? If so, what are their equations?
- b) Evaluate the first and second derivatives step by step and factor the results.
- c) Find the zeros of the first and second derivatives by hand.
- d) Make sign charts for  $f$ ,  $f'$  and  $f''$  and above them draw a stick figure graph and concavity icons as appropriate.
- e) Identify the exact (and 2 decimal place accurate values of the) coordinates  $(x,y)$  of points on the graph of  $f$  which are critical points, indicating whether they are local maxima or minima or neither.
- f) Identify the exact (and 2 decimal place accurate values of the) coordinates  $(x,y)$  of points on the graph of  $f$  which are points of inflection.
- g) Make a roadmap plot of  $f$  showing all of these points including intercepts labeled by approximate 2 decimal place values of their coordinates, identifying critical points by  $C(x,y)$  and inflection points by  $I(x,y)$ , and write in the equations of any horizontal asymptotes, identifying them as well.

**Instructions.** Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers exact (no decimal approximations, if possible). After you are done, sign and date:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date: