

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. **BOX** final short answers. **LABEL** parts of problem. Keep answers exact (no decimal approximations, if possible). [See long instructions on website.]

Each calculation requiring the evaluation of a derivative on this test must be supported by a detailed "step-by-step" derivation of the final simplified form of the derivative formula. [Every such derivation can (and should!) be checked with technology.] No collaboration! Come see me if you are confused, but not at the last minute.

① $y = \ln(\arcsin(\frac{x}{2}))$. Evaluate $\frac{dy}{dx}$ and $\frac{dy}{dx}|_{x=1}$ exactly, simplified as much as possible.

② $y = 15(e^{\frac{x}{30}} + e^{-\frac{x}{30}}) - 25$. A telephone line hangs between two poles 12m apart in the shape of a catenary whose equation is given, where x and y are measured in meters. Find the slope where it meets the right pole ($x=6$), exactly and to 3 decimal place accuracy.

③ Find the points on the curve $y = 2x^3 + 3x^2 - 36x + 7$ (i.e., both coordinates (x_i, y_i))
 a) where the tangent line is horizontal, b) where the tangent line is parallel to the line $y = -24x$. Make a completely labeled rough sketch of the curve and these tangent lines (preferably from a technology window appropriately chosen).

④ $f(x) = x^2(2-x)^3$, $\frac{1}{2} \leq x \leq \frac{3}{2}$. a) Find the points (x, y) on the graph of this function at which it assumes its absolute maximum and absolute minimum on the given interval. Sketch all of this information in a graph based on a technology plot. [Use the critical pt/endpoint procedure and give both exact values as rational numbers p/q and decimal approximations to 2 decimal place accuracy.]

⑤ a) Find an equation of the tangent to the curve $y = e^x$ that passes through the origin. [Hint: first write the equation of the tangent line to the curve at an arbitrary point $x=a$, then determine a by requiring that $(0,0)$ satisfy that equation.] Your final result should be exact (no decimals) and simplified to slope-intercept form.
 b) Make a completely labeled sketch illustrating what you have found.
 c) Use the linear approximation to e^x at $x=1$ to approximate $e^{1.1}$ (decimal result).

⑥ a) Find the equation of the tangent line to the curve $x^2 + 4xy + y^2 = 13$ at the point $(2,1)$. Where does the tangent line intersect the x and y axes?
 b) Can you make a technology plot which confirms your work? If so, sketch your plot labeling the key points.

⑦ $y = \frac{\sin(mx)}{x}$, $m > 0$ constant. a) Evaluate and simplify the derivative.
 b) What is the simplest explicit equation that determines the value of x at which the tangent line is horizontal?
 c) Solve that equation numerically when $m=2$, explaining how you do so, for the smallest positive value of x that satisfies it.

⑧ As the sun rises, the angle of elevation of the sun with respect to the horizon is increasing at a rate of $.25 \text{ rad/hr}$ (roughly 180° during 12 hours, no?).

How fast is the shadow cast by a 50 ft flagpole decreasing when the angle is 60° ?

- Introduce any variables needed and indicate the rate of change given and the rate of change to be found using the proper $d?/d?$ or $d?/d?$ notation. Make a diagram illustrating all of this.
- State the relationship between the corresponding variables.
- Solve the problem, giving an exact value and a decimal approximation with 3 significant digits.
- Answer the word problem question with a sentence that does not use any symbols you have introduced.
- At this rate how many inches will the shadow move in 1 minute?

⑨ A computer driven experimental vehicle on I-80 crossing the Salt Flats makes a 40 mi trip in 10 min. Its position in miles along the highway from its starting location is $s(t) = 20(1 - \cos(6\pi t))$, where t is the time in hours from the start.

- Make a completely labeled s versus t plot (with consistent time units) describing this trip.
- Evaluate the average velocity for this trip and draw in the corresponding secant line in your diagram for part a).
- Evaluate the velocity function and make a completely labeled rough sketch of it.
- When does the speedometer readout equal the average velocity? Give your times in minutes. Support your answer with detailed calculations.
- When is the vehicle traveling at its fastest speed?
- Evaluate the acceleration function. When during this trip is the acceleration or deceleration greatest? What is its absolute value, with units?

⑩ A paper water fountain cup has the shape of an inverted cone with equal height and radius. Use the differential approximation to estimate the percentage increase in its capacity (volume) if the dimensions increase by 2%.

Take home test: If you use (computer) technology to check your work, print out your worksheet with comments, handwritten if necessary, labeled by problem, and attach it to your test after your hand work.

When you have completed the exam, please read and sign the dr bob integrity pledge and attach it to your answer sheets (staple take home test) as a cover page, first side face up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date: