MAT1500-04/06 04F Test 1B   Print Name (Last, First) ______________________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers exact (no decimal approximations, if possible). [See long instructions on reverse].

1. a) Make a diagram showing a typical (curving) function graph $y=f(x)$, labeling the axes, and identify a point "a" on the x-axis and connect it to the corresponding point on the graph and then over to the y-axis. Then locate a nearby point "a+h" to its right and repeat. Then connect the two points on the graph (secant line segment) and draw in the remaining side(s) of the natural triangle with this hypotenuse. Label the two other sides of this triangle by their (signed) lengths, write down an expression for the secant line slope m_{sec} and its limit m_{tan}.

b) Use this limit definition to evaluate the tangent line slope for $f(x)=\frac{2}{x-1}$ at $x=a$.

c) Use your result to write the equation of the tangent line to f at $x=2$. What are the x and y intercepts of the tangent line?

d) Plot f and this tangent line with technology and make a rough sketch of what you see, labeling tickmarks with key values. Does it look right? (If not, say no!). Explain.

2. $f(x)=\begin{cases} \frac{2}{x-1}, & x\geq 2 \\ 2+\sqrt{2-x}, & x<2 \end{cases}$ Is this function continuous on the entire real line? Explain why or why not using limit notation. Make a rough sketch of the function.

3. $f(x)=\frac{4-x}{3+x}$ a) Evaluate using reasoning, not technology readout:

(i) $\lim_{x\to\infty} f(x)$ (ii) $\lim_{x\to-\infty} f(x)$ (iii) $\lim_{x\to-3^+} f(x)$ (iv) $\lim_{x\to-3^-} f(x)$ (v) $\lim_{x\to-3} f(x)$

b) Does f have any horizontal asymptotes? If so, what are their equations and in which directions ($x\to\infty$ or $x\to-\infty$) are they asymptotes?

c) Does f have any vertical asymptotes? If so, what are their equations?

d) Where does f cross the x-axis?

e) Does $f(x)=-1$ have a solution? Show your work.

f) Use all of the above information to sketch the graph of f. Does it look right compared to your technology plot? What single window $x=a..b$, $y=c..d$ shows all of this behavior clearly? (This is a judgement call – no exact number ranges can be said to be the one right answer, but rough values of the window can be given.)

4. Evaluate these limits, explaining by reasoning out what happens in the limit.

a) $\lim_{x\to 0} \frac{1}{2+e^{-3/x^2}}$ b) $\lim_{x\to\infty} \frac{1}{2+e^{-3/x^2}}$

c) Would the reasoning you used in a) and b) still work if 3 were replaced by 300 or 0.003? Explain.