MAT2705-01/04 03S Quiz 7 Print Name (Last, First) ______________________

last first

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations).

① $Ly = (D^3+1)y = (D+1)(D^2-D+1)y = (D+1)(D-[\frac{1}{2}+i\frac{\sqrt{3}}{2}])(D-[\frac{1}{2}-i\frac{\sqrt{3}}{2}])y$

Write down the general solution of $Ly=0$ (in explicitly real form).

② $Ly = (D^3-2D^2-D+2)y = (D-1)(D-2)(D+1)y$

Solve the initial value problem $Ly=0$, $y(0)=1$, $y'(0)=0=y''(0)$ in three steps:

a) Write down the general solution.

b) Write down the system of equations needed to determine the arbitrary constants.

c) Write down the corresponding augmented matrix and use technology to reduce it completely (write down the result) and solve it. Be sure your final answer $y=\ldots$ is fully evaluated and boxed.

① characteristic equation: $r^3+1 = (r+1)(r-(\frac{1}{2}+i\frac{\sqrt{3}}{2}))(r-(\frac{1}{2}-i\frac{\sqrt{3}}{2})) = 0$

roots: $r=-1$, $\frac{1}{2}\pm i\sqrt{3}/2$ → frequency ω

exps: e^{-x}, $e^{x/2}e^{\pm i\sqrt{3}/2}$

general solution: $\boxed{y = c_1 e^{-x} + e^{x/2}(c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x)}$

② a) characteristic equation: $(r^3-2r^2-r+2) = (r-1)(r-2)(r+1) = 0$

roots: $r=1,2,-1$

exps: e^{x}, e^{2x}, e^{-x}

general solution: $\boxed{y = c_1e^{x} + c_2e^{2x} + c_3e^{-x}}$

b) $y = c_1e^x + c_2e^{2x} + c_3e^{-x}$

$y' = c_1e^x + 2c_2e^{2x} - c_3e^{-x}$

$y'' = c_1e^x + 4c_2e^{2x} + c_3e^{-x}$

$$\begin{aligned} y(0) &= c_1+c_2+c_3 = 1 \\ y'(0) &= c_1+2c_2-c_3 = 0 \\ y''(0) &= c_1+4c_2+c_3 = 0 \end{aligned} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & 1/3 \end{bmatrix}$$

$c_1 = 1$

$c_2 = -1/3$

$c_3 = 1/3$

← backsub

$\boxed{y = e^{x} - \frac{1}{3}e^{2x} + \frac{1}{3}e^{-x}}$