Usual instructions, Label, box, etc. Show all relevant work.

Mat2705 03S Quiz 5 Print Name (Last,First): ______________________ $/10$

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> with(linalg):
> A:=matrix([[5,6,1,2,5,6,1,0],[3,4,3,4,7,8,0,1]]);
```

$$A := \begin{bmatrix} 5 & 6 & 1 & 2 & 5 & 6 & 1 & 0 \\ 3 & 4 & 3 & 4 & 7 & 8 & 0 & 1 \end{bmatrix}$$

a) Give the sequence of row operations (in order) needed to row reduce this matrix using the notation: **mulrow**(%,row#,coef), **addrow**(%,row#, torow#,coef), **swaprow**(%,row#,row#) and give the final **rref** reduced matrix.

b) Let C be the 2x4 matrix formed by deleting the last 4 columns from A and let B be the 2x2 matrix:

```
> B:=matrix([[4,-6],[-3,5]]);
```

$$B := \begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix}$$

Evaluate the matrix product BC by hand.

c) Based on your row reduction work so far and not by doing any further work, solve the following system associated with the first 3 columns of A and explain how you obtained your solution from your previous work:

```
>   sys:={5*x+6*y=1,3*x+4*y=3};
```

$$sys := \{5\,x + 6\,y = 1, 3\,x + 4\,y = 3\}$$

d) Write out the system of 2 linear equations in 4 variables $x1, x2, x3, x4$ which has the matrix form $C\,X = 0$, where the 0 is the 2x1 zero matrix. Again from your previous row reduction result, write out the equivalent linear system after the row reduction and solve it for X, writing the result in column matrix form, and then pulling apart the solution: $X = t_1\,C1 + t_2\,C2 + \ldots$ by expressing it in terms of the column matrices of coefficients of the parameters.

a) Only the first two columns of A affect the choice of reduction steps:

$$\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix} \xrightarrow{\text{mulrow}(\%,1,1/5)} \begin{bmatrix} 1 & 6/5 \\ 3 & 4 \end{bmatrix} \xrightarrow{\text{addrow}(\%,1,2,-3)} \begin{bmatrix} 1 & 6/5 \\ 0 & 2/5 \end{bmatrix} \xrightarrow{\text{mulrow}(\%,2,5/2)} \begin{bmatrix} 1 & 6/5 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{addrow}(\%,2,1,-6/5)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -7 & -8 & -11 & -12 & 2 & -3 \\ 0 & 1 & 6 & 7 & 10 & 11 & -3/2 & 5/2 \end{bmatrix}$$ by technology.

b)
$$\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 5 & 6 & 1 & 2 \\ 3 & 4 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 4\cdot5-6\cdot3 & 4\cdot6-6\cdot4 & 4-6\cdot3 & 4\cdot2-6\cdot4 \\ -3\cdot5+5\cdot3 & -3\cdot6+5\cdot4 & -3+5\cdot3 & -3\cdot2+5\cdot4 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -14 & -16 \\ 0 & 2 & 12 & 14 \end{bmatrix}$$

c)
$$\text{rref}\left(\begin{bmatrix} 5 & 6 & 1 \\ 3 & 4 & 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 6 \end{bmatrix} \rightarrow \boxed{\begin{matrix} x = -7 \\ y = 6 \end{matrix}}$$

first 3 columns of rref(A), rewrite corresponding system eqns.

d)
$$\underbrace{\begin{bmatrix} 5 & 6 & 1 & 2 \\ 3 & 4 & 3 & 4 \end{bmatrix}}_{C}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} 5x_1 + 6x_2 + x_3 + 2x_4 = 0 \\ 3x_1 + 4x_2 + 3x_3 + 4x_4 = 0 \end{matrix} \rightarrow \text{rref}(C) = \begin{bmatrix} 1 & 0 & -7 & -8 \\ 0 & 1 & 6 & 7 \end{bmatrix}$$ same as above.

augmented matrix: $\begin{bmatrix} 5 & 6 & 1 & 2 & 0 \\ 3 & 4 & 3 & 4 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -7 & -8 & 0 \\ 0 & 1 & 6 & 7 & 0 \end{bmatrix}$

$$\begin{matrix} x_1 - 7x_3 - 8x_4 = 0 & \rightarrow & x_1 = 7t_1 + 8t_2 \\ x_2 + 6x_3 + 7x_4 = 0 & \rightarrow & x_2 = -6t_1 + 7t_2 \\ x_3 = t_1 & \rightarrow & x_3 = t_1 \\ x_4 = t_2 & \rightarrow & x_4 = t_2 \end{matrix} \rightarrow \bar{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7t_1 + 8t_2 \\ -6t_1 + 7t_2 \\ t_1 \\ t_2 \end{bmatrix} = t_1\begin{bmatrix} 7 \\ -6 \\ 1 \\ 0 \end{bmatrix} + t_2\begin{bmatrix} 8 \\ 7 \\ 0 \\ 1 \end{bmatrix}$$