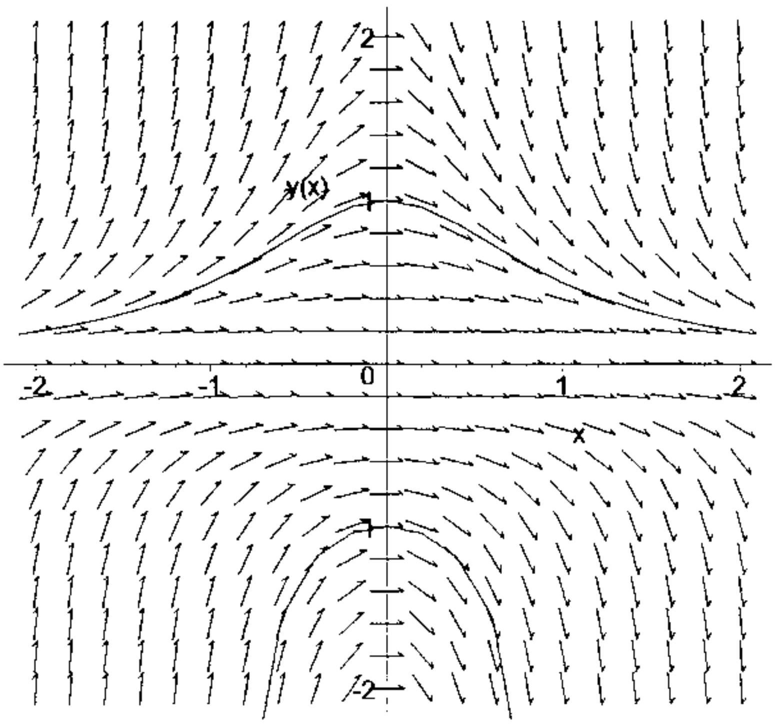
MAT2705-01/04 03S Quiz 2 Print Name (Last, First) ____________________ /10

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers. Always simplify expressions.

D.E. $\frac{dy}{dx} = -2xy^2$ I.C. (i): $y(0)=1$ I.C. (ii): $y(0)=-1$



consistent?

a) Draw in the two solution curves corresponding to the two initial conditions.

b) Find the (almost) general solution of this D.E.

c) Impose the I.C. (i) on your solution stating your final result for y.

d) Repeat for the I.C. (ii).

e) Check that your solution c) satisfies the D.E.

f) The missing solution $y=0$ divides the solution family into the upper and lower half plane solutions. Assuming your solutions c) and d) are respectively typical of these two kinds of solutions, what statements can you make about the existence of horizontal or vertical asymptotes for these curves?

a) see above right.

b) $\frac{dy}{dx} = -2xy^2 \rightarrow \frac{dy}{y^2} = -2x\,dx \rightarrow \int y^{-2}\,dy = -2\int x\,dx \rightarrow \frac{y^{-1}}{-1} = -2\left(\frac{x^2}{2}\right) + C \rightarrow$

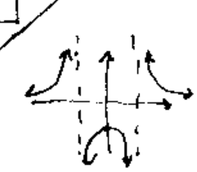
$-\frac{1}{y} = -x^2 + C \rightarrow \frac{1}{y} = x^2 - C \rightarrow \boxed{y = \frac{1}{x^2 - C}}$

c) $y(0)=1 \leftrightarrow x=0, y=1 \longrightarrow 1 = \frac{1}{0^2-C} = -\frac{1}{C} \rightarrow C=-1 \rightarrow \boxed{y = \frac{1}{x^2-(-1)} = \frac{1}{x^2+1}}$

d) $y(0)=-1 \leftrightarrow x=0, y=-1 \longrightarrow -1 = \frac{1}{0^2-C} = -\frac{1}{C} \rightarrow C=1 \rightarrow \boxed{y = \frac{1}{x^2-1}}$

e) $y = \frac{1}{x^2+1} = (x^2+1)^{-1} \rightarrow \frac{dy}{dx} = -1(x^2+1)^{-2}(2x) = -\frac{2x}{(x^2+1)^2}$

$\frac{dy}{dx} = -2xy^2 \longrightarrow \frac{-2x}{(x^2+1)^2} = -2x\left(\frac{1}{x^2+1}\right)^2 = \frac{-2x}{(x^2+1)^2}$ ✓



f) Looking at the formulas $\lim_{x\to\pm\infty} \frac{1}{x^2 \pm 1} = 0$ so both have $y=0$ as a hor. asympt, while only $y = \frac{1}{x^2-1}$ has vertical asymptotes (at $x=\pm1$). Looking at our graph we don't see the horizontal asymptote for the lower solution — because we can't reach the parts of the solution across the vertical asymptotes where y becomes POSITIVE. Thus our impression from the graph has missed this remaining family of curves! (which can't be obtained continuously from initial data on the y axis).