

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers. Always simplify expressions.

① $f(x) = \frac{3}{x}$ a) Find $f'(a)$ using the limit definition of a derivative.

b) From part a) find an equation describing the tangent line to the graph $y = 3/x$ at $x=3$, simplifying your result to the standard slope-intercept form $y = mx + b$.

② Life expectancy has improved dramatically in the 20th century. The table gives values of $E(t)$, the life expectancy at birth (in years) of a male born in the year t in the USA. Interpret and estimate the value $E'(1980)$.

t	1970	1980	1990
E(t)	67.1	70.1	71.8

$$\begin{aligned} \text{① a) } f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{\frac{3}{a+h} - \frac{3}{a}}{h} = \lim_{h \rightarrow 0} \frac{\frac{3a - 3(a+h)}{a(a+h)}}{h} = \lim_{h \rightarrow 0} \frac{\overset{0}{3a} - \overset{0}{3a} - 3h}{a(a+h)h} \\ &= \lim_{h \rightarrow 0} \frac{-3h^1}{\underset{1}{h} a(a+h)} = \lim_{h \rightarrow 0} \frac{-3}{a(a+h)} = \frac{-3}{a(a+0)} = \boxed{-\frac{3}{a^2}} \end{aligned}$$

$$\begin{aligned} \text{b) } \underbrace{f(3) = \frac{3}{3} = 1}_{\text{pt } (3,1)}, \quad f'(3) = \underbrace{-\frac{3}{3^2} = -\frac{1}{3}}_{\text{slope}} \quad \left. \begin{aligned} & y - 1 = -\frac{1}{3}(x - 3) \\ & y = 1 - \frac{1}{3}(x - 3) \\ & = 1 - \frac{x}{3} + 1 \\ & = 2 - \frac{x}{3} \end{aligned} \right\} \end{aligned}$$

$$\boxed{y = 2 - \frac{x}{3}}$$

t	E(t)			
②	1990	71.8	1.7	$\rightarrow \frac{1.7}{10} = 0.17$
10 <	1980	70.1		\swarrow
10 <	1970	67.1	3.0	$\rightarrow \frac{3.0}{10} = 0.30$
			adjacent sec line slopes	

$$\frac{0.17 + 0.30}{2} = \frac{0.47}{2} = \boxed{0.235 \approx E'(1980)}$$

avg is
tan line slope

This is the rate of change of life expectancy in 1980: 0.235 years life expectancy per year. In other words you would expect the life expectancy in 1981 to be about this much longer.