MAT1505-05/08 015 Quiz 9 Print Name (Last, First) ____________________ /10

Usual instructions.

① a) Write down the formula for the Taylor series of a function $f(x)$ about center $x=1$ (ie a power series in powers of $(x-1)$).

b) Evaluate the first 4 nonzero terms of the Taylor series for $f(x)=x^{1/2}$ about $x=1$.

② $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ a) Write out the first 4 nonzero terms.

b) If $12° \approx .2094395$ radians, how many terms are needed to evaluate $\sin 12°$ to 5 decimal place accuracy? Explain.

c) Evaluate $\sin 12°$ to 5 decimal place accuracy (your result should only have 5 decimal places).

① a) $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)\,(x-1)^n}{n!}$ $\left(= f(1) + f'(1)(x-1) + \frac{1}{2}f''(1)(x-1)^2 + \frac{1}{6}f^{(3)}(1)(x-1)^3 + \cdots\right)$

b) $f(x) = x^{1/2}$ $f(\ ) = 1$

$f'(x) = \frac{1}{2}x^{-1/2}$ $f'(1) = \frac{1}{2}$

$f''(x) = \frac{1}{2}\left(-\frac{1}{2}\right)x^{-3/2}$ $f''(1) = \frac{1}{2}\left(-\frac{1}{2}\right) = -\frac{1}{4}$

$f'''(x) = \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^{-5/2}$ $f^{(3)}(1) = \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{3}{8}$

$$f(x) = 1 + \frac{1}{2}(x-1) + \frac{1}{2}\left(-\frac{1}{4}\right)(x-1)^2 + \frac{1}{6}\left(\frac{3}{8}\right)(x-1)^3 + \cdots$$

$$= \boxed{1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 + \cdots = x^{1/2}}$$

② $\sin x = \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \cdots$

$\sin(.2094395) \approx .2094395$
$- .0015311739 33$ } $\approx .2079083261 \to \boxed{.20791}$

$+ .3358239890 \times 10^{-5}$ $< .5\times10^{-5}$ $(n=2)$ alternating series error

$- .3507354069 \times 10^{-8}$

$+ \cdots$

(error estimated by **next** term **after** truncating the series)