MAT 1505-05/08 015 QUIZ 7 Print Name (Last, First) ______________ /10

Show all work on this sheet, including indications of mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation/syntax. Label parts, $\boxed{\text{box}}$ find short answers.

① $S = \sum_{n=1}^{\infty} \frac{1}{n^{11}}$ (eleventh power)

a) Justify the convergence of this infinite series.

b) Write out the 5th partial sum exactly and give its decimal equivalent.

c) Estimate the truncation error R_5 using the integral remainder approach.

d) How many terms do you need to get 10 decimal place accuracy? Show all work.

e) On the basis of d), evaluate S to 10 decimal place accuracy.

① a) This is a p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ with $p = 11 > 1$ and so converges.

[since the corresponding integral converges: $\int_1^{\infty} x^{-11}\,dx = \frac{x^{-10}}{-10}\Big|_1^{\infty} = \frac{1}{10}$]

b) $\boxed{S_5 = 1 + \frac{1}{2^{11}} + \frac{1}{3^{11}} + \frac{1}{4^{11}} + \frac{1}{5^{11}} \approx 1.000494184}$

c) $R_5 < \int_5^{\infty} x^{-11}\,dx = \frac{x^{-10}}{-10}\Big|_5^{\infty} = -\frac{1}{10x^{10}}\Big|_5^{\infty} = \text{"}\cancel{-\frac{1}{10\cdot\infty^{10}}}\text{"} + \frac{1}{10\cdot 5^{10}} = \boxed{\frac{1}{10\cdot 5^{10}}}$

$\boxed{\approx .1024\times 10^{-7}}$

d) $R_n < \int_n^{\infty} x^{-11}\,dx = -\frac{1}{10x^{10}}\Big|_n^{\infty} = \frac{1}{10\cdot n^{10}} < \frac{1}{2}\times 10^{-10}$

take reciprocals: $10n^{10} > 2\cdot 10^{10}$

$n^{10} > \frac{2}{10}10^{10}$

$n > \left(\frac{2}{10}10^{10}\right)^{1/10} = \left(\frac{2}{10}\right)^{\frac{1}{10}} 10 = \left(\frac{1}{5}\right)^{\frac{1}{10}}\cdot 10$

≈ 8.513

so $\boxed{n=9}$ is the least number of terms

e) $S_9 = \sum_{n=1}^{9} \frac{1}{n^{11}} \approx \boxed{1.000494188}$

of course if we are working only with 10 significant digits, we should also worry about computer truncation error...